UNIT- III

SOLUTIONS OF EQUATIONS AND EIGEN VALUE PROBLEMS

IMPORTANT FORMULAE

NEWTON RAPHSON METHOD

$$\phi(x_n) = x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0,1,2,\dots, \quad |\phi'(x)| < 1$$

UNIT- V

NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

TAYLOR'S SERIES

$$y_{n+1} = y_n + \frac{h}{1!} y_n' + \frac{h^2}{2!} y_n'' + \frac{h^3}{3!} y_n''' + \cdots$$

EULER'S METHOD

$$y_{n+1} = y_n + h f(x_n, y_n)$$

MODIFIED EULER'S METHOD

$$y_{n+1} = y_n + \frac{1}{2} h \left[f(x_n, y_n) + f(x_n + h, y_n + h f(x_n, y_n)) \right]$$

RUNGE-KUTTA METHODS OF FOURTH ORDER

$$k_1 = h f(x_1, y_1)$$
$$k_2 = h f\left(x + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$
$$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$
$$k_4 = h f(x_1 + h, y_1 + k_3)$$
$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

PREDICTOR - CORRECTOR METHODS

PREDICTOR FORMULA :

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

CORRECTOR FORMULA

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

BOUNDARY VALUE PROBLEMS

$$y'_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y''_i = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

UNIT- IV IMPORTANT FORMULAS.

① Lagrange Interpolation formula.

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 .$$

②. Newton's divided difference interpolations.

$$f(x) = f(x_0) + (x-x_0) \cdot f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2)$$

$$+ \cdots + (x-x_0)(x-x_1) \cdots (x-x_{n-1}) \cdot f(x_0,x_1,\cdots x_n).$$

③. Newton's forward Interpolation formula for equal intervals.

$$y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \cdot \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \cdots$$

Here $u = \frac{x-x_0}{h}$

④ Newton's Backward interpolation formula.

$$y(x) = y_n + \frac{v}{1!} \nabla y_n + \frac{v(v+1)}{2!} \nabla^2 y_n + \frac{v(v+1)(v+2)}{3!} \nabla^3 y_n + \cdots$$

Here $v = \frac{x-x_n}{h}$.

⑤. Newton's forward difference formula.

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \cdots \right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

(6). Newton's Backward difference formula.

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \cdots\right]$$

(7). Trapezoidal rule.

$$I = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

(8). Simpson's 1/3 rule,

$$I = \frac{h}{3}\left[(y_0+y_n) + 2(y_1+y_3+y_5+\cdots) + 4(y_2+y_4+\cdots)\right]$$

(9). Simpson's 3/8 rule.

$$I = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\cdots+y_{n-1}) + 2(y_3+y_6+\cdots)\right].$$