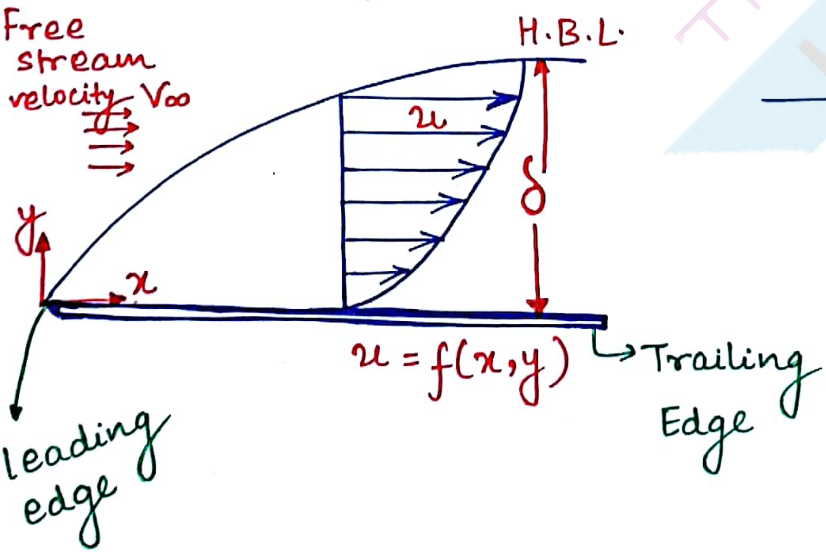
CONVECTION

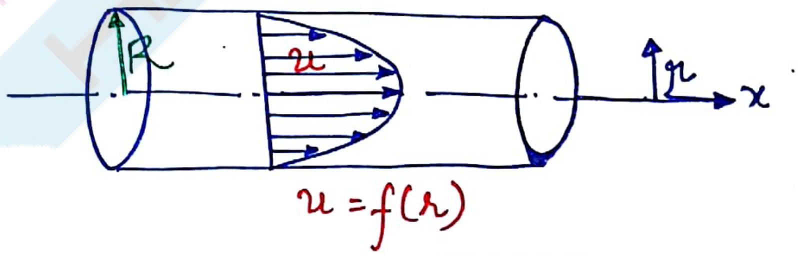
Note :- In any convection Heat transfer problem it is req. to obtain the convective heat transfer coefficient 'h' for a given boundary conditions prevailing b/w Body and fluid and hence to obtain the convection heat transfer rate b/w them from newton's law of cooling i.e.

$$\boxed{q = h A \Delta T_{conv.}}$$

Convection

- **Forced Convection H.T.**
  velocity is evident.
  - Flow over Flat plates
  - Flow through pipes/Ducts (Internal flows)
- **Free (OR) Natural Convection**
  No velocity evident but the flow occurs naturally due to Buyancy forces arising out of density changes of fluid.





$$\boxed{\text{Mass flow Rate} = \dot{m} = \rho \times \pi R^2 \vec{V}_{mean} \ \text{kg/sec}}$$

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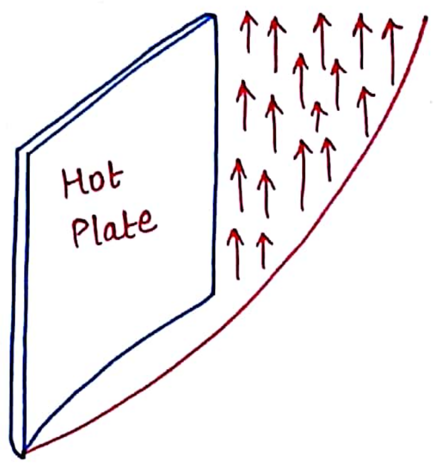
Free convection



$$\frac{P}{\rho \downarrow} = RT\uparrow$$

* FORCED CONVECTION HEAT TRANSFER :-

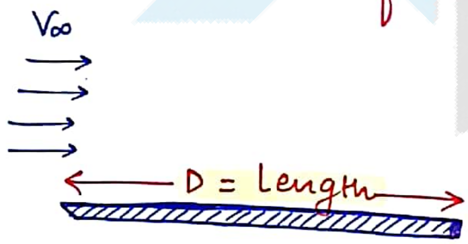
In any forced convection heat transfer,

$$h = f(\vec{V}, D, \underbrace{\rho, \mu, C_p, K}_{\text{Thermophysical Properties of fluid.}})$$

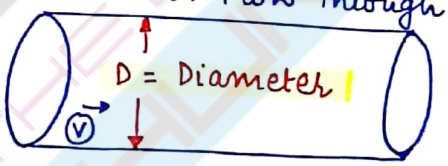
$\vec{V}$ = velocity of fluid (m/sec)

D = characteristic Dimension of Body

(Dimension of Body i.e. used in the calculation of dimensionless No.s).

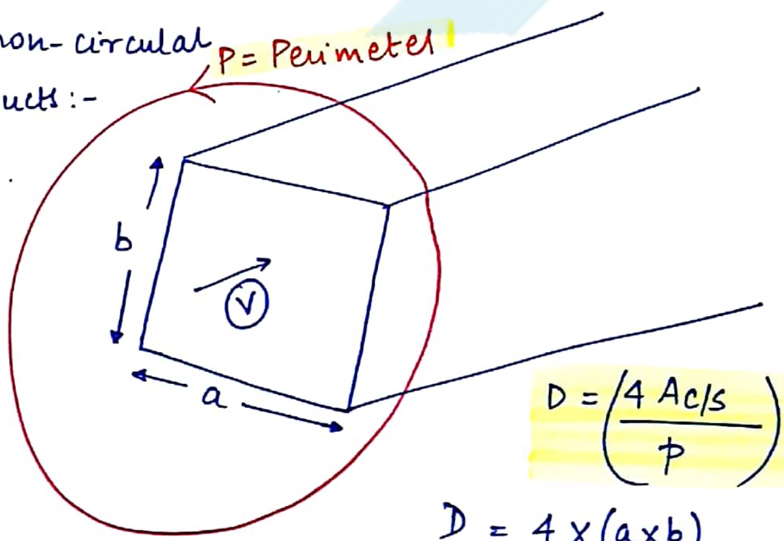


For Flow through pipes



$$D = \frac{4 \times \frac{\pi}{4} D^2}{\pi D} = D$$

For non-circular Ducts :-



$$D = \left(\frac{4 A_{c/s}}{P}\right)$$

$$D = \frac{4 \times (a \times b)}{2(a+b)}$$

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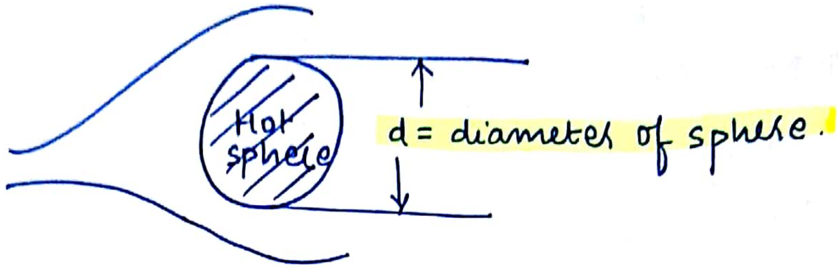

* from Buckingham-π Theorem of dimensional analysis which states that if there are Total 'n' no. of variables in any functional relationship both dependent and Independent and if all the variables put together contain 'm' no. of fundamental dimensions, then the functional relationship among the variables can be expressed in terms of ('n - m') no. of dimensionless π-Terms.

Here $n = 7$

$m = 4(M, L, T, \theta)$ → Tempr.

$\therefore$ from Theorem,

no. of dimensionless π-terms $= 7 - 4 = 3$.

Let the π-terms be π_1, π_2 and π_3.

To get $\pi_1 = ?$ $\pi_2 = ?$ and $\pi_3 = ?$

- $D \rightarrow L$
- $V \rightarrow LT^{-1}$
- $\rho \rightarrow ML^{-3}$
- Pa-sec $= \mu \Rightarrow ML^{-1}T^{-1}$

- $\dfrac{\cancel{M}LT^{-2}}{\cancel{M}\theta} = \dfrac{Nm}{kg\,K} = \dfrac{J}{kg\,K} = Cp \Rightarrow L^2T^{-2}\theta^{-1}$

- $\dfrac{MLT^{-2}}{T\theta} = \dfrac{N\cancel{m}}{sec\,\cancel{m}K} = \dfrac{J}{sec\,mK} = \dfrac{W}{mK} = K \rightarrow MLT^{-3}\theta^{-1}$

- $\dfrac{W}{m^2K} = h \rightarrow MT^{-3}\theta^{-1}$.

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choose 'm' no. of Repeating variables in such a way that

① All of them put together contain all the fundamental dimensions.

② They themselves should NOT form a dimensionless group.

∴ Select h, V, D, ρ as Repeating variables.

Then

$$\pi_1 = (h^{a_1} \cdot V^{b_1} \cdot D^{c_1} \cdot \rho^{d_1})\,\mu.$$

$$\pi_2 = (h^{a_2} \cdot V^{b_2} \cdot D^{c_2} \cdot \rho^{d_2})\,C_p.$$

$$\pi_3 = (h^{a_3} \cdot V^{b_3} \cdot D^{c_3} \cdot \rho^{d_3})\,K.$$

Now, To get π_1 :-

$$M^0L^0T^0\theta^0 = (MT^{-3}\theta^{-1})^{a_1}(LT^{-1})^{b_1}(L)^{c_1}(ML^{-3})^{d_1} \times ML^{-1}T^{-1}.$$

For Mass 'M' :- $0 = a_1 + d_1 + 1$

For ~~Mass~~ length 'L' :- $0 = b_1 + c_1 - 3d_1 - 1$

For Time 'T' :- $0 = -3a_1 - b_1 - 1$

For Tempr. 'θ' :- $0 = -a_1$

$a_1 = 0$
$b_1 = -1$
$c_1 = -1$
$d_1 = -1$

$$\Rightarrow \boxed{\pi_1 = \frac{\mu}{VD\rho}}$$

To get π_2 :- $M^0L^0T^0\theta^0 = (MT^{-3}\theta^{-1})^{a_2}(LT^{-1})^{b_2}(L)^{c_2}(ML^{-3})^{d_2} \times L^2T^{-2}\theta^{-1}$

For mass 'M' :- $0 = a_2 + d_2$

For length 'L' :- $0 = b_2 + c_2 - 3d_2 + 2$

For time 'T' :- $0 = -3a_2 - b_2 - 2$

For ~~time~~ Temp. 'θ' :- $0 = -a_2 - 1$

$\Rightarrow$ $a_2 = -1$
$b_2 = 1$
$c_2 = 0$
$d_2 = 1$

$$\Rightarrow \boxed{\pi_2 = \left(\frac{\rho V C_p}{h}\right)}$$

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To get π_3 :-

$$M^0 L^0 T^0 \theta^0 = (MT^{-3}\theta^{-1})^{a_3} \cdot (LT^{-1})^{b_3} \cdot (L)^{c_3} \cdot (ML^{-3})^{d_3} (MLT^{-3}\theta^{-1})$$

$a_3 = -1$
$b_3 = 0$
$c_3 = -1$
$d_3 = 0$

$\therefore \pi_3 = \frac{K}{hD}$

hence,

$$\pi_1 = \frac{\mu}{VD\rho}$$

$$\pi_2 = \frac{\rho V c_p}{h}$$

$$\pi_3 = \frac{K}{hD}$$

Let

$$\pi_4 = \frac{1}{\pi_1} = \frac{VD\rho}{\mu} = \text{Reynold's No.} = (Re)$$

$$\pi_5 = \frac{1}{\pi_3} = \frac{hD}{K} = \text{Nusselt No.} = (Nu)$$

also → (dimensionless heat transfer coefficient)

$$\pi_6 = \frac{\pi_1 \pi_2}{\pi_3} = \frac{\mu c_p}{K} = \text{Prandtl No.} = (Pr)$$

→ all μ, c_p, K → Thermophy. properties

$$\pi_7 = 1/\pi_2 = \frac{h}{\rho V c_p} = \text{Stanton No.} = (St)$$

hence, important in GATE

$\therefore$ In any forced convection H.T.,

From Theorem,

$$Nu = f(Re, Pr)$$

$$\frac{hD}{K} = f\left(\frac{VD\rho}{\mu}, \frac{\mu c_p}{K}\right)$$

our aim to get this here outside.

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* PHYSICAL SIGNIFICANCE OF DIMENSIONLESS NO.'s IN forced Convection Heat Transfer :-

① **Reynold's No (Re)** :- Reynolds No. is defined as the Ratio b/w inertia forces and viscous forces to which a flowing fluid is subjected to

$$Re. = \frac{I.F.}{V.F.}$$

$$Re = \frac{VD\rho}{\mu}$$

$$Re = \frac{VD}{\left(\frac{\mu}{\rho}\right)}$$

$$Re = \frac{VD}{\nu}$$

where ν = Kinematic viscosity of fluid in $\frac{m^2}{sec}$

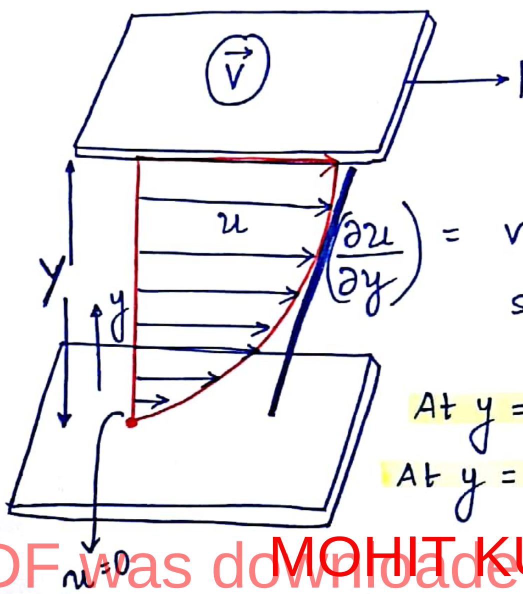
$$\nu = \left(\frac{\mu}{\rho}\right)$$

ν ($\frac{1}{\rho}$ V.) signifies Momentum Diffusion Rate through fluid layers.

✓ viscosity is a property of fluid by virtue of which there is a resistance offered by one layer of the fluid over its adjacent layer against the relative motion between them.

✓ Newton's Law of Viscosity :-

$F_{net} = m \cdot a$, $a \to 0$



$\left(\frac{\partial u}{\partial y}\right)$ = velocity gradient (or) Rate of shear strain (or) Rate of angular deformation.

At $y = 0 \Rightarrow u = 0$

At $y = Y \Rightarrow u = V$

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The law states :-

$$\text{shear stress} \propto \left(\frac{\partial u}{\partial y}\right)$$

$$\tau \propto \left(\frac{\partial u}{\partial y}\right)$$

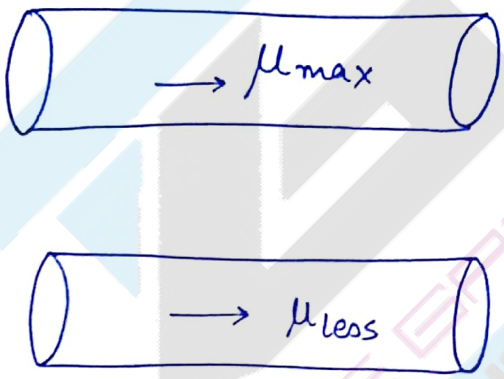
$$\boxed{\tau = \mu\left(\frac{\partial u}{\partial y}\right) \text{ Pascal}}$$

property of a fluid. ← μ = Dynamic viscosity of fluid in Pa-sec

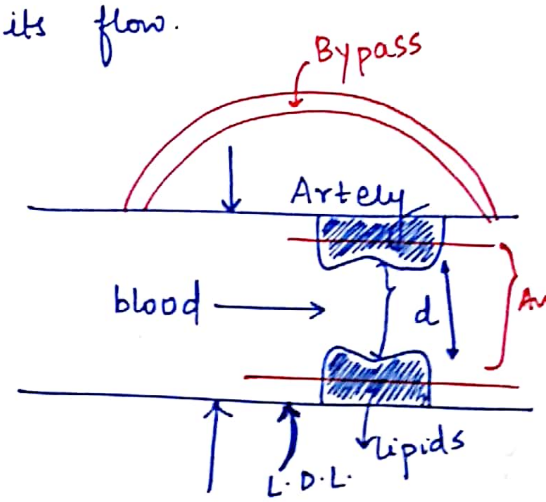
$$\mu_{air} = 18\times10^{-6} \text{ Pa-sec}$$

(At Room conditions)

*



Note :-
A fluid having more viscosity need to be supplied with more amount of energy for a given volume flow rate in a given pipe diameter because it suffers more frictional resistance against its flow.



$$\uparrow h_f \downarrow = \frac{fLv^2}{2gd\downarrow} \quad \mu\downarrow$$

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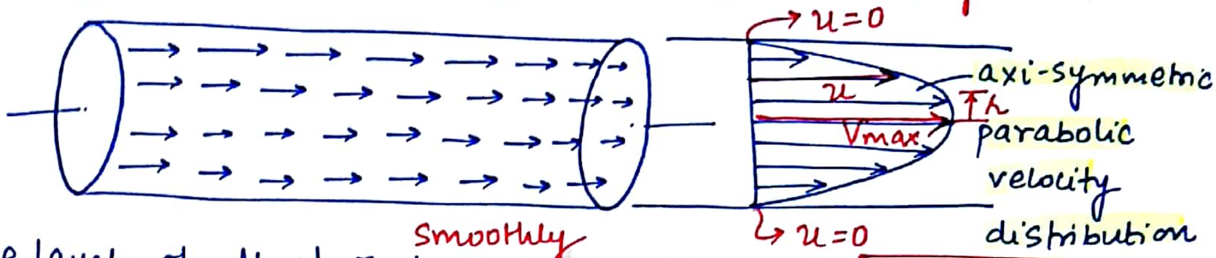
Re is the criteria To tell whether the fluid flow is laminar or turbulent. for incompressible flow through pipes as well as ducts, (171)

- If Re < 2000 (lower critical Re. No.) ⇒ Flow is laminar flow/Viscous flow.

English $\boxed{\text{LAMINA} \rightarrow \text{moving Planes}}$ → No intermixing



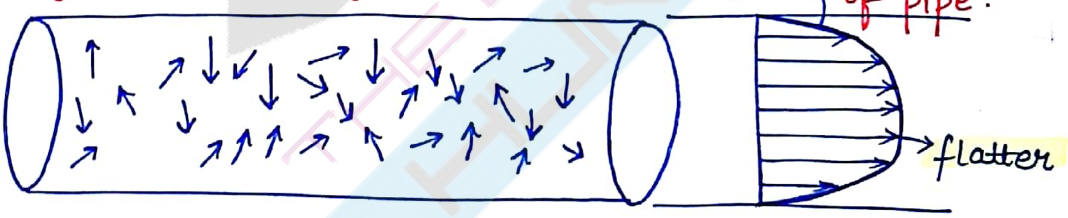
one layer of fluid Just smoothly slides over its adjacent layer without any intermingling (or) Intermixing of fluid particles among fluid layers.

$\boxed{u = f(r)}$

$\boxed{\text{Ex}}$:- ① The flow of highly viscous lubricant oil at relative low velocities in a small diameter pipe is generally laminar.

- If Re > 4000 (upper critical Re. No.) ⇒ Flow is turbulent

logrithmic velocity distribution at a given cross-section of pipe.



$u = f(r)$

Continuous intermingling (OR) Intermixing of fluid particles among fluid layers with chunks of fluid Mass jumping from one layer to another layer carrying more Mass, Momentum and energy with them.

$\boxed{\text{Ex}}$:- The flow of any gas even at low velocities inside a pipe or duct are generally turbulent flows.

Generally, more often we come across turbulent flows during flow through pipes.

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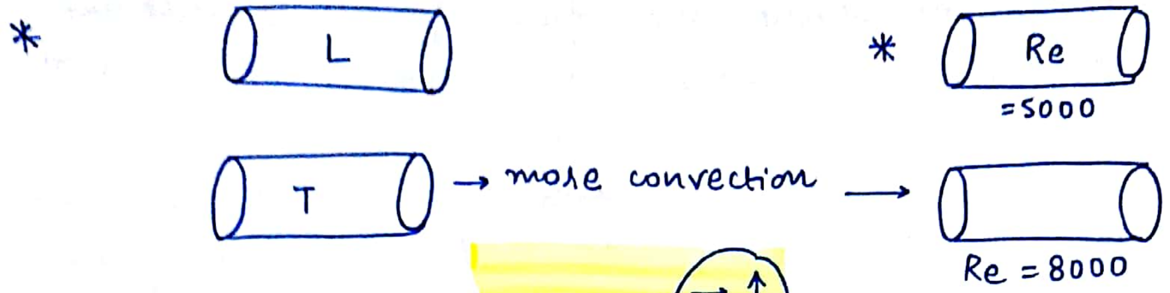
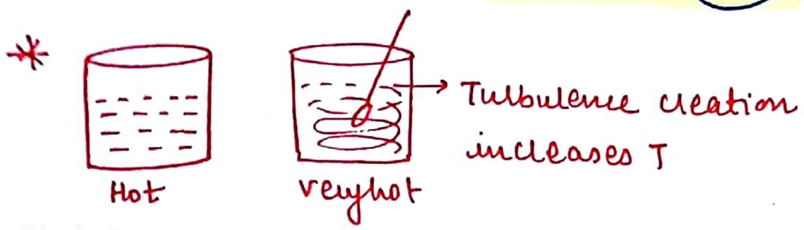
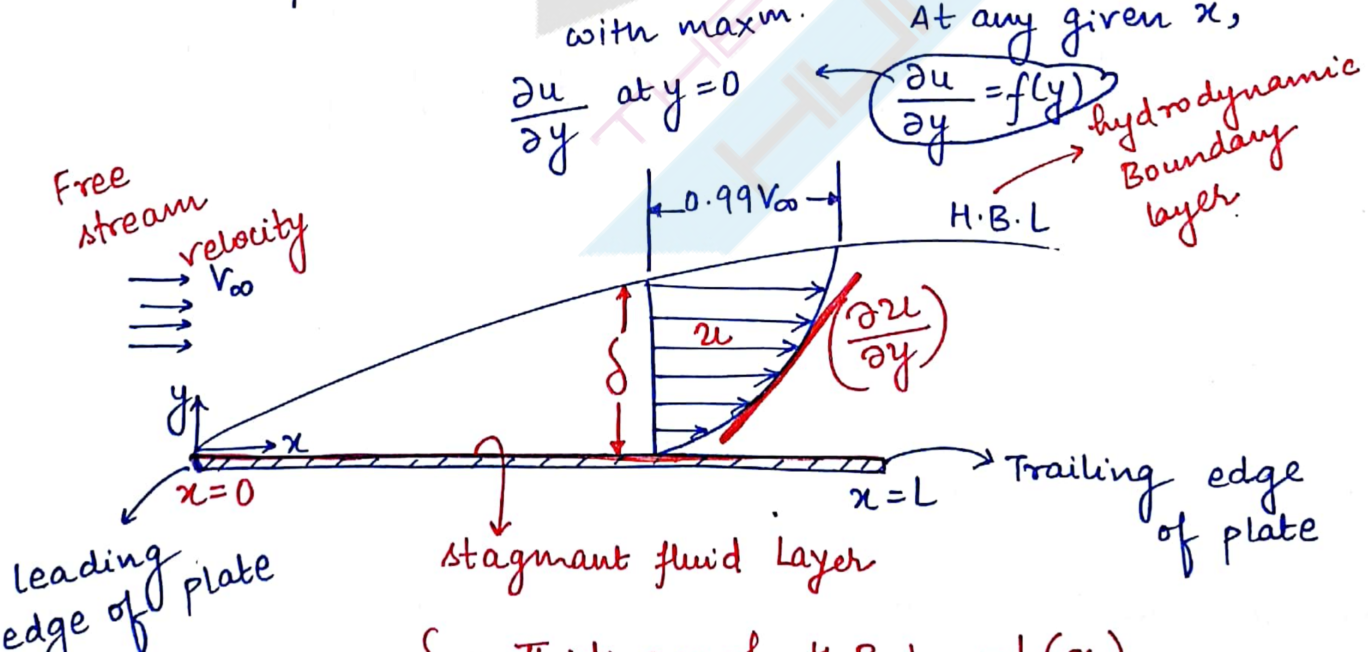
* 

To ↑ Re, $\vec{V}$ ↑

* 

✓ For any given fluid in a given pipe, the convective h.T. coefficient's in turbulence flow's are always greater than those in laminar flows. also, for a given turbulent flow of a fluid in a given pipe, as the Reynold's no. increases (that is by ↑ing the velocity of fluid), the convective H.T. coefficient 'h' also increased.

* **FOR Incompressible Flow over Flat plates:-**



δ = Thickness of H.B.L. $= f(x)$

$u = f(x, y)$

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Hydrodynamic Boundary layer is defined as a thin region formed over the flat plate inside which velocity gradients are seen in the normal direction to the plate (flat). These velocity gradients are formed due to the viscous nature or rather "due to" momentum diffusion through the fluid layers in the normal direction to the plate (y - direction). Outside this HBL, everywhere free stream velocities prevail (existing) that is no viscous influence felt. (173)

The Boundary conditions of HBL are :-

at any x, measured from leading edge of plate,

At any given x,

At $y = 0 \Rightarrow u = 0$

At $y = \delta \Rightarrow u = 0.99\, V_\infty$

At $y = \delta \Rightarrow \frac{\partial u}{\partial y} = 0$

At $y = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0$

Also, at any given x,

$\left(\frac{\partial u}{\partial y}\right) = f(y)$

with maxm. value of $\frac{\partial u}{\partial y}$ at $y = 0$.

$\delta = f(x)$

26/9/2016

Let local Reynold's No. $= \boxed{Re_x = \left(\frac{V_\infty x \rho}{\mu}\right) = \frac{V_\infty x}{\nu}}$

The flow over flat plate is laminar,

If $\boxed{Re_x < 5 \times 10^5}$

The flow over flat plate is Turbulent,

if $\boxed{Re_x > (6.5 \text{ to } 7 \times 10^5)}$

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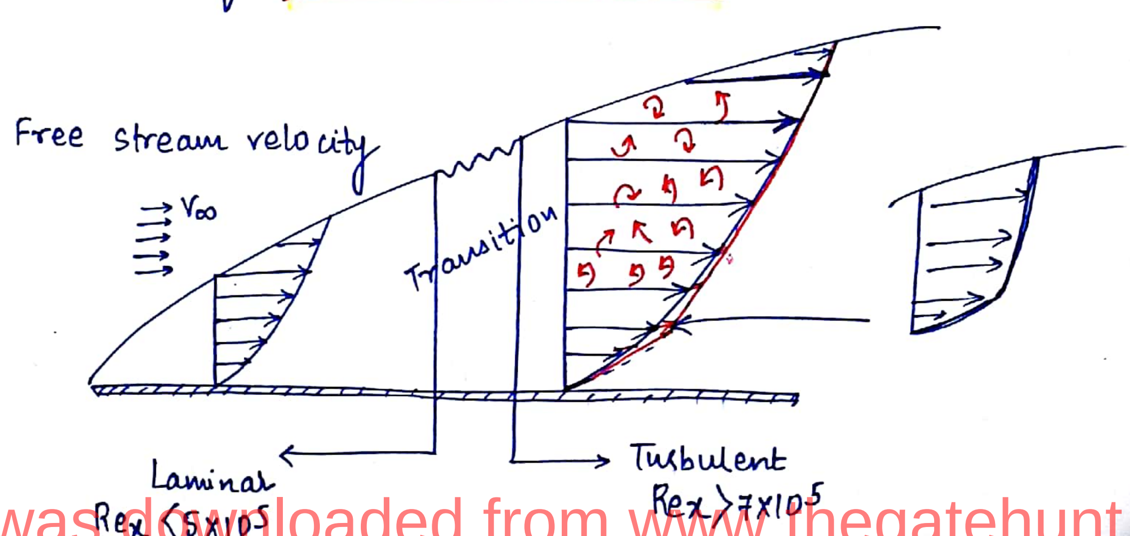

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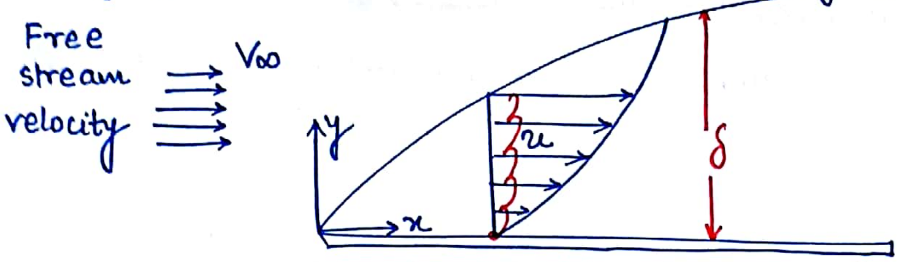
NOTE :- Most commonly we come across laminar flows during flow over flat plates and turbulent flows during flow through pipes or Ducts.

for laminar flow over flat plate, the velocity distribution within the hydrodynamic Boundary layer is given as :-
at any given x, measured from leading edge of plate,



$$\boxed{\frac{u}{V_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$

where δ = HBL Thickness

δ_h $= f(x)$

$$\boxed{\delta = \frac{4.64x}{\sqrt{Re_x}}} \quad \text{(OR)} \quad \boxed{\frac{5.0x}{\sqrt{Re_x}}}$$

(Blausiu's solution)

$$\boxed{Re_x = \left(\frac{V_\infty x}{\nu}\right)}$$

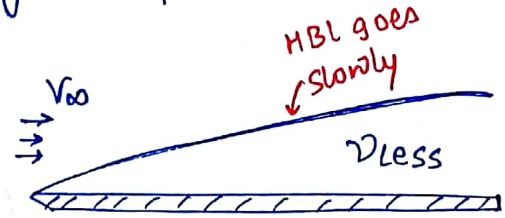
Now, $\delta = \dfrac{5.0x}{\sqrt{\dfrac{V_\infty x}{\nu}}}$ ← kinematic viscosity (property of fluid)

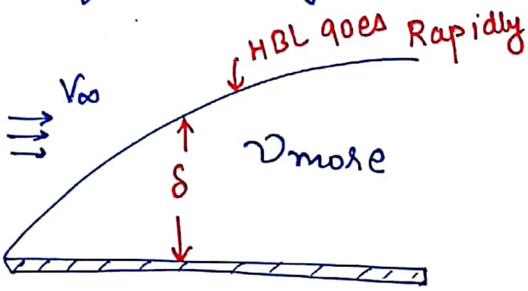
$\Rightarrow$ $\boxed{\delta \propto x^{1/2}}$

we may also conclude that,

$\Rightarrow$ $\boxed{\delta \propto \sqrt{\nu}}$

Note :- Therefore a fluid having more kinematic viscosity shall form thicker HBL as compared to the fluid having lesser K.V. for a given free stream velocity.





K.V. (ν) ⟶ Momentum diffusivity

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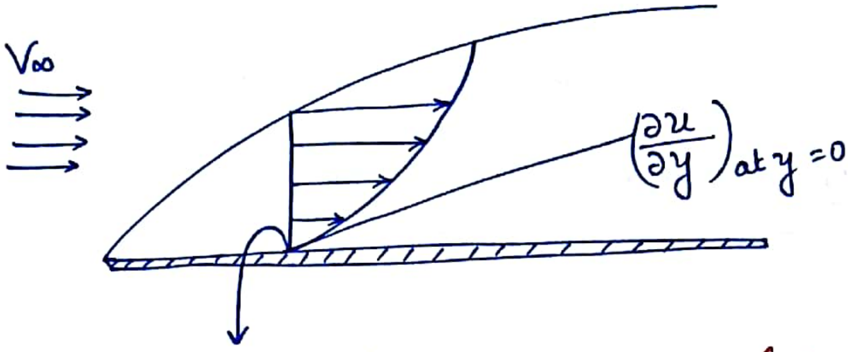
Newton's law of viscosity for flow over flat plate :-



τ_ω = local wall shear stress (at any given x)

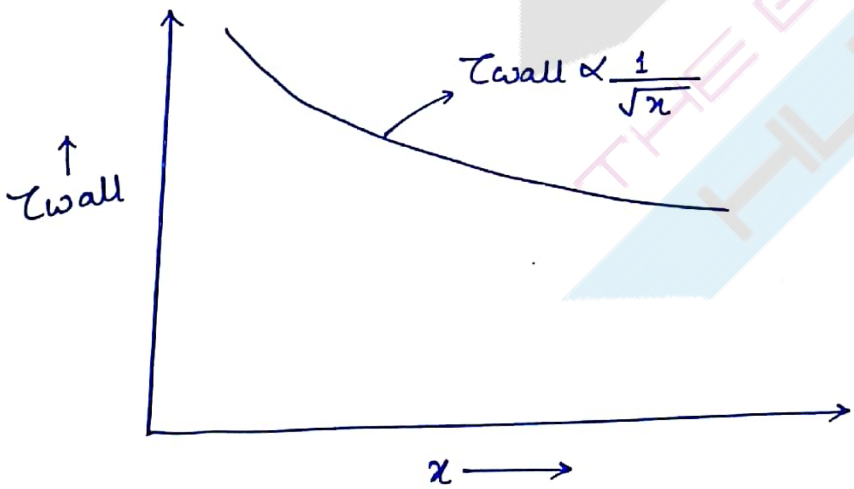
$$= \mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0} = \left(\mu V_\infty \frac{3}{2\delta}\right) \text{Pascal}$$

{after differentiating
$$\frac{u}{V_\infty} = \frac{3}{2}(y/\delta) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$
and put $y=0$ }

$$\boxed{\tau_\omega = \left(\mu V_\infty \frac{3}{2\delta}\right) \text{Pascal}}$$

Since $\delta \propto x^{1/2}$

$$\Rightarrow \boxed{\tau_{wall} \propto x^{-1/2}}$$



Note :- This local wall shear stress physically indicates the frictional drag (or) the drag resistance exerted by the fluid on the flat plate.

$$\boxed{\underset{\text{(average shear stress)}}{\overline{\tau}} = 2 \times (\tau_{wall})_{at\ x=L}}$$

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$$\text{Drag force on Plate} = \tau \times \text{Area of Plate}$$

(2) Nusselt No. (Nu) :-

$$Nu = \frac{hD}{K_{fluid}} = \frac{(D/KA)}{\left(\frac{1}{hA}\right)}$$

Nu resembles Biot No.

$$\frac{hD}{K_{fluid}} \qquad \frac{hs}{K_{solid}}$$

$$Nu = \frac{hD}{K_{fluid}} = \frac{(D/KA)}{(1/hA)} = \frac{\text{Conduction Resistance offered by fluid if it were stationary}}{\text{surface convective Resistance}}$$

since fluids being bad conductors of heat,

Numerator > Denominator

$\therefore$ $\boxed{Nu \text{ is always} > 1.}$

Nu is also called as dimensionless H.T. coefficient.

(3) Prandtl No. (Pr) :-

Prandtl is the only dimensionless no. which is the property of a fluid defined as the ratio between kinematic viscosity of the fluid and its thermal diffusivity.

$$Pr = \frac{K.V.}{T.D.} = \left(\frac{\nu}{\alpha}\right) = \frac{\frac{\mu}{\rho}}{\frac{K}{\rho C_p}} = \left(\frac{\mu C_p}{K}\right)$$

$\mu \to$ Pa-sec, $C_p \to$ J/kgK, $K \to$ W/mK

Pr of air → 0.65 to 0.73

Pr of water → 2 to 6

Pr of liquid Metals like Hg is very low since their 'K' value is very high.

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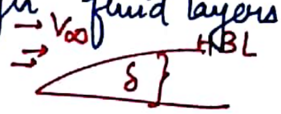
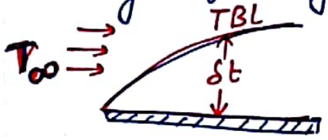
For lubricating oils (engine oils),

Pr may go upto 100.

(ν) → signifies Momentum Diffusivity Rate through fluid layers.
(K.V.)

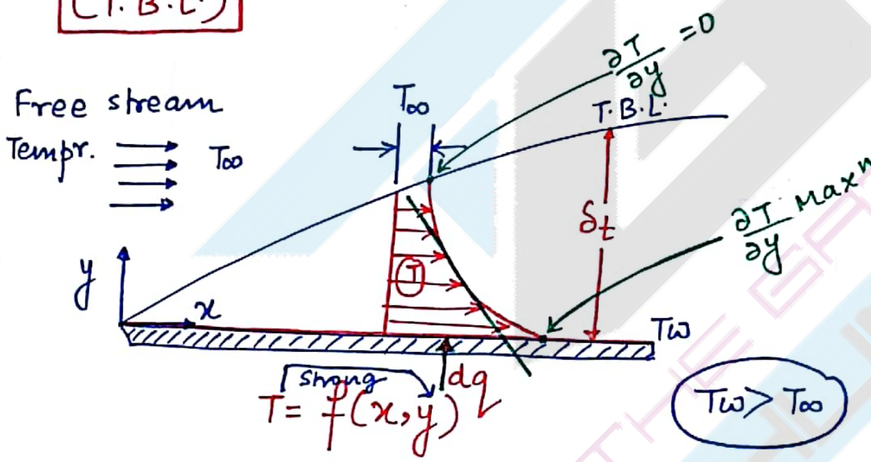
α (T.D.) → signifies Heat Energy Diffusion Rate through fluid layers.





Prandtl No. signifies the relative magnitudes of Momentum diffusion Rate and heat Energy diffusion Rate that are occuring through the fluid layers in the normal direction to the plate simultaneously in the Respected Boundary layers i.e. H.B.L. and T.B.L.
Thermal Boundary layer.

* THERMAL BOUNDARY LAYER – (T.B.L.)



δ_t = thickness of T.B.L. = $f(x)$.

The thermal Energy transported by Moving fluid = $\dot{m} C_p T$.

The actual mechanism of Heat transfer in any convection is first heat gets conducted through the stagnant fluid layer at $y = 0$ and then while conducting through the fluid layers in the normal direction to the plate (y-direction). Heat is also simultaneously transported / convected away by the ~~[illegible]~~ moving fluid layers entering & leaving from the Boundary layer in the form of $\dot{m} C_p T$ (J/sec)

Defn. of T.B.L. :- Just similar to H.B.L. inside which velocity gradients are seen in the normal dirn. to the plate, Thermal Boundary level is also a thin Region inside which tempr. gradients are present in the normal dirn. to the plate. These Tempr. gradients

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are formed due to the heat transfer b/w the plate & flowing fluid.

The Boundary conditions of T.B.L. are :-

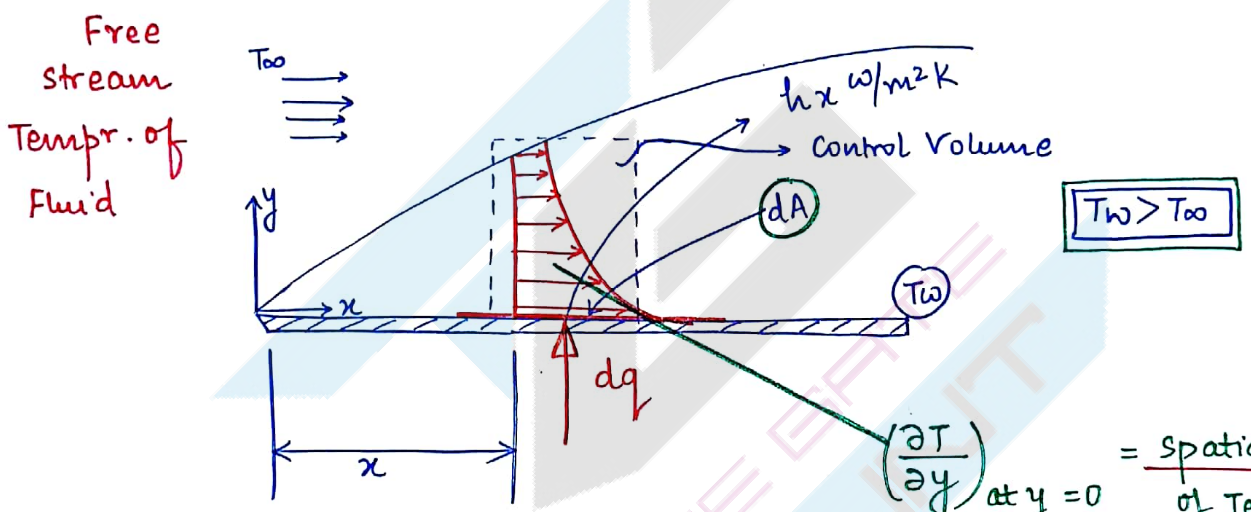
At any x, measured from leading edge of plate,

$$\left[\begin{array}{l} \text{At } y=0 \Rightarrow T=T_w \\ \text{At } y=\delta_t \Rightarrow T=T_\infty \\ \text{At } y=\delta_t \Rightarrow \dfrac{\partial T}{\partial y}=0 \\ \text{At } y=0 \Rightarrow \dfrac{\partial^2 T}{\partial y^2}=0 \end{array}\right]$$

at any given x, $\dfrac{\partial T}{\partial y}=f(y)$

with maximum value of $\dfrac{\partial T}{\partial y}$ at $y=0$.

Energy Balance for a differential Control volume of T.B.L. are :-



$\left(\dfrac{\partial T}{\partial y}\right)_{at\ y=0}$ = spatial gradient of Tempr. within the fluid at $y=0$

Consider a differential control volume of Area 'dA' at a distance of 'x' measured from the leading edge of the plate. The C.V. is extending from the plate upto the edge of the Boundary layer.

Writting the energy Balance for steady state conditions of C.V.,

Heat conducted into the C.V. through stagnant fluid layer at $y=0$ = Heat convected from the hot plate at T_w to free stream fluid at T_∞.

Fourier's law of Conduction Newton's law of cooling

$$-k_f \cancel{dA}\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = h_x \cancel{dA}\,(T_w - T_\infty)$$

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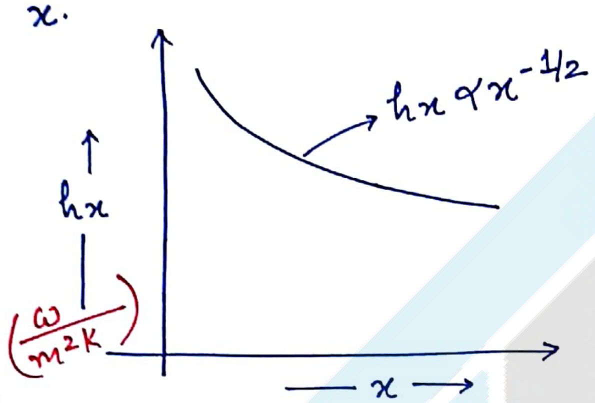
$$\Rightarrow \boxed{h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}}{(T_w - T_\infty)} \quad \frac{\text{watt}}{m^2\text{-}K}}$$

--→ {$\left(\frac{T}{y}\right)$ fn. → find करो}

Local convective heat transfer coefficient. (H.T.)

Since the spatial gradient of Tempr. within the fluid at $y=0$ keeps on decreasing with increase of x, the local convective H.T. coefficient 'h_x' also must be decreasing with increase of x.



Gate 2009

Q.> A coolant fluid at 30°C flows over an isothermal flat plate which is maintained at a Tempr. of 100°C. The Boundary layer Tempr. distribution within the fluid at some location of x measured from the leading edge is $T = 30 + 70\,e^{(-y)}$ where y is the distance measured in the normal dirn. to plate. If Thermal conductivity of fluid is 1W/mK. The local convective H.T. coefficient h_x at that location will be ______?

(a) 10 W/m²K

(b) 5 W/m²K

(c) 1 W/m²K

(d) 2 W/m²K

Sol> $\frac{\partial T}{\partial y} = 70 + 70e^{-y}(-1)$

1 ✓

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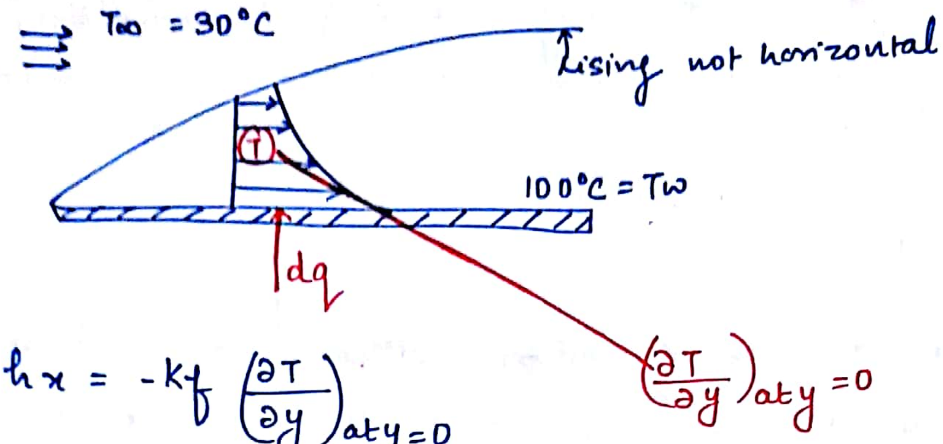
SIR



$$\therefore h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{T_w - T_\infty}$$

$$h_x = \frac{-1 \times \frac{\partial}{\partial y}\left(30 + 70e^{-y}\right)_{at\, y=0}}{(100 - 30)}$$

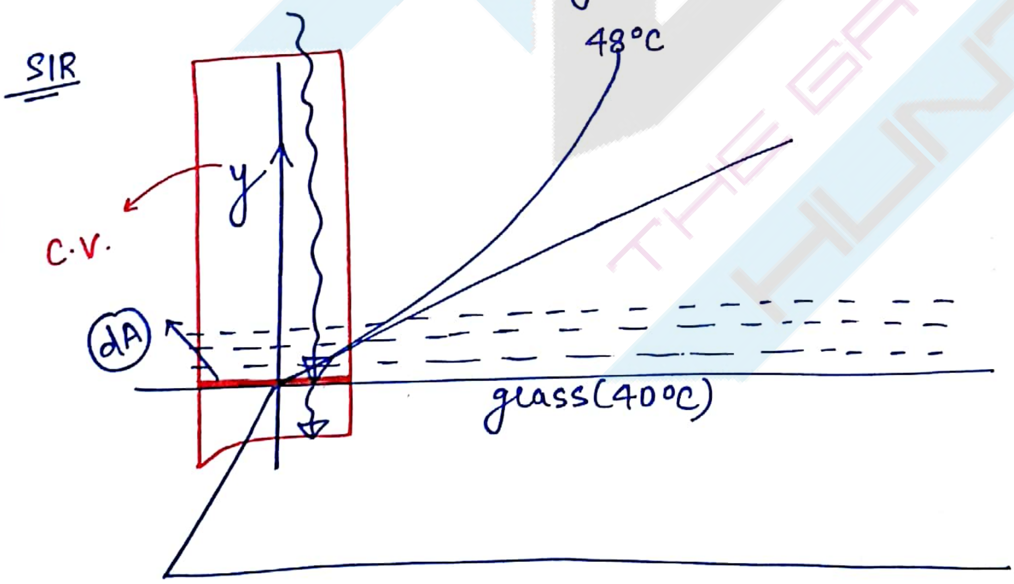
$$h_x = 1 W/m^2K$$

Pg 84
(4) (5) (4) $T_\infty = 48°C$

$T_w = 40°C$ $K_w = 0.6\ W/mK$ $\frac{dT}{dy} = 1 \times 10^4 \frac{K}{m}$

$K_g = 1.2\ W/mK$

SIR



Consider a differential Control Volume of Area 'dA' which is extending from the free stream water at 48°C and well into the glass plate.

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writting the energy Balance for steady state conditions of differential c.v., 181

Heat ~~conducted~~ convected from the free stream water at 48°c to stagnant water layer at 40°c } = Heat Conducted through stagnant water layer at Interface } = Heat Conducted through glass plate.

Newton's

fourier's

fourier's

$$\Rightarrow h\cancel{dA}(48-40) = K_w \cancel{dA}\left(\frac{dT}{dy}\right)_{\text{within water at interface}}$$

$$= K_{glass}\cancel{dA}\left(\frac{dT}{dy}\right)_{\text{within glass at Interface}}$$

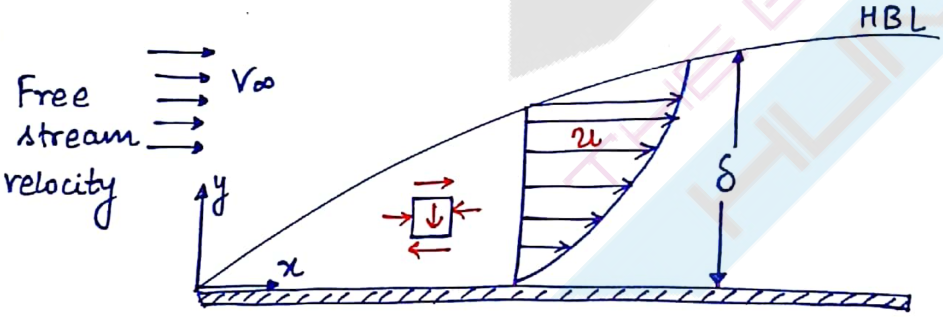
$$\Rightarrow h = \frac{0.6\times1\times10^4}{(48-40)}$$

$$= 750\ W/m^2K$$

$$\left(\frac{dT}{dy}\right)_{\text{within glass at interface}} = \frac{0.6\times(1\times10^4)}{1.2}$$

$$= 0.5\times10^4\ K/\text{metre}$$

* Momentum Equation of H.B.L. :- (F.M.)



Assume :- ①

Steady, 2D, Incompressible flow ($\rho = c$)

$u = f(x,y)$ $\qquad v = f(x,y)$

② Unit depth of element (⊥r to Plane of fig.)

③ Real fluid flow.

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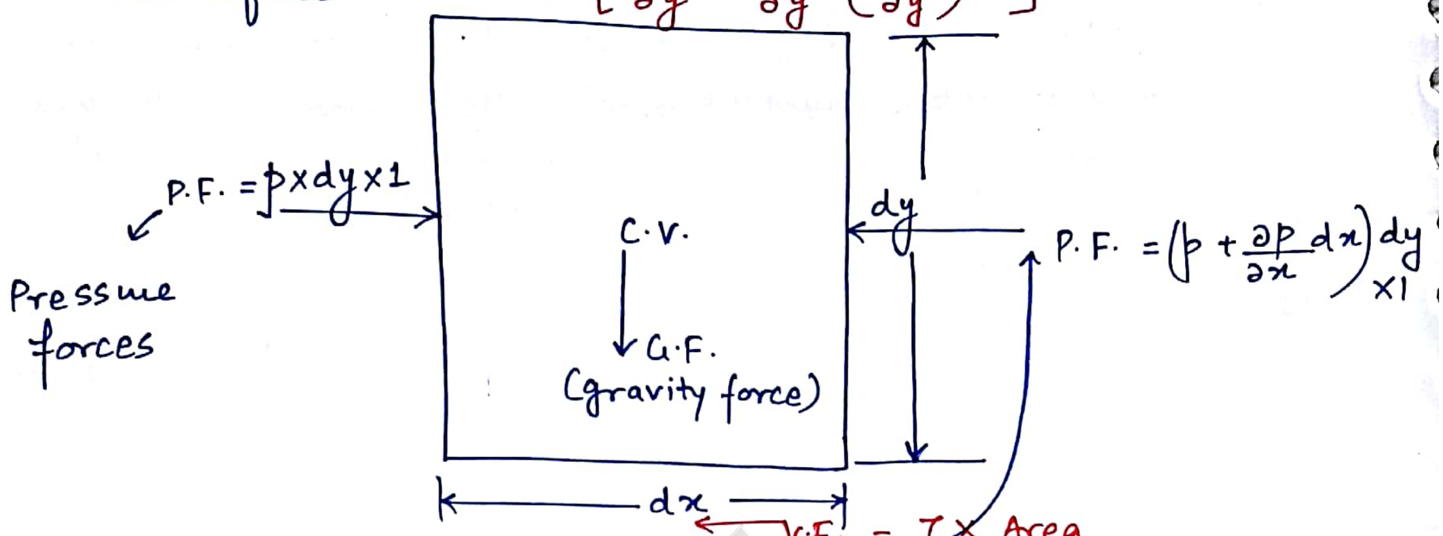
viscous force ← $\vec{V.F.} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right](dx \times 1)$



$$P.F. = \left\{p + \left(\frac{\partial P}{\partial x}\right)dx\right)dy \times 1$$

$$= \mu \times \left(\frac{\partial u}{\partial y}\right) \times (dx \times 1).$$

Newton's IInd Law of Motion :-

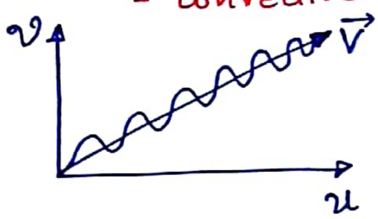
$$\overset{\oplus}{\overrightarrow{\Sigma F_x}} = m\,\overset{\oplus}{\overrightarrow{a_x}}$$ (Conservation of linear Momentum)

Net algebraic sum of all the forces acting on fluid element along x- direction = Rate of change of linear momentum of element along x- direction.

The Resulting Momentum Eqn. of H.B.L. is :- K.V. (Kinematic viscosity)

$$\boxed{\underbrace{\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)}_{\text{convective accl}^r\text{. of fluid along } x.} = \left(\frac{\mu}{\rho}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\left(\frac{dP}{dx}\right)}$$

→ Navier - Stokes eqn. of Motion along x.



For flow over flat plates,

$$\boxed{\frac{dP}{dx} = 0}$$

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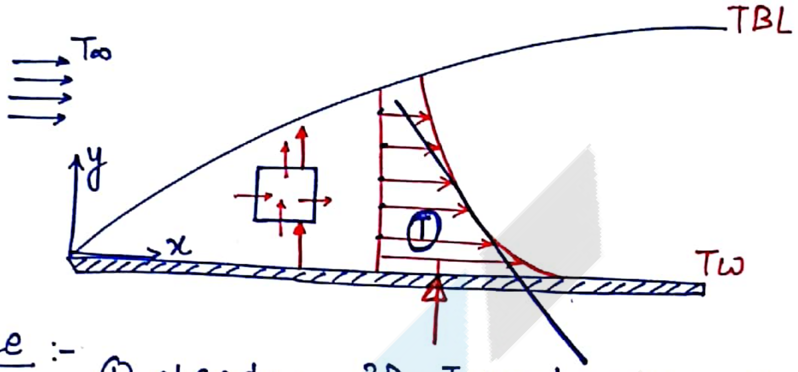
and Put $\frac{\mu}{\rho} = \nu$ = K. V. of fluid m^2/sec.

$$\Rightarrow \boxed{u\left(\frac{\partial u}{\partial x}\right) + v\frac{\partial u}{\partial y} = \underset{K.V.}{\nu} \cdot \frac{\partial^2 u}{\partial y^2}}$$

Momentum equation of H.B.L.

* ENERGY EQUATION OF T.B.L. :-



Assume :-

① steady, 2D, Incompressible flow ($\rho = c$)

$u = f(x,y)$ $v = f(x,y)$ $T = f(x,y)$ strong weak

② Constant fluid properties (ρ, μ, c_p, k)

③ Unit depth of element (⊥ar to plane of fig.)

④ Negligible conduction along x- direction.

The Thermal energy transported / convected by flowing fluid = $\dot{m} c_p T$ J/sec

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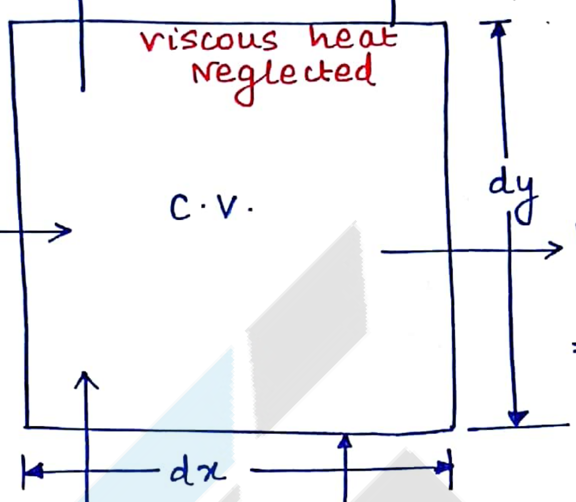
Heat convected through Top face = $\boxed{\rho(dx\times 1)\left(v+\frac{\partial v}{\partial y}\cdot dy\right)C_p\left(T+\frac{\partial T}{\partial y}\cdot dy\right)}$

Heat conducted through Top face = $\boxed{-k_f(dx\times 1)\left[\frac{\partial T}{\partial y}+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)dy\right]}$

Six Energies



Heat convected through Left face = $\boxed{\rho(dy\times 1)uC_pT}$

Heat convected through Right face = $\boxed{\rho(dy\times 1)\left[u+\frac{\partial u}{\partial x}\cdot dx\right]C_p\left[T+\frac{\partial T}{\partial x}\cdot dx\right]}$

Heat convected through bottom face = $\boxed{\rho(dx\times 1)vC_pT}$

Heat conducted through Bottom face = $\boxed{-k_f(dx\times 1)\frac{\partial T}{\partial y}}$

Writting the Energy Balance for steady state conditions of C.V. we get,

Heat Conducted through Bottom face
+
Heat Convected through Left face
+
Heat Convected through Bottom face

=

Heat conducted thro. Top face
+
Heat convected thro. Right face
+ Heat convected through Top face

The Resulting energy eqn. of T.B.L. is :-

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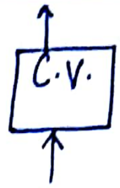
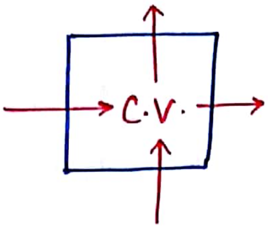
$$\underbrace{\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)}_{\text{Net heat convected to C.V.}} = \underbrace{\left(\frac{K}{\rho C_p}\right)\frac{\partial^2 T}{\partial y^2}}_{\text{Net heat conducted out of C.V.}}$$





But $\frac{K}{\rho C_p} = \alpha$ = T.D. of fluid (m^2/sec)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

(Tempr. field) Energy eqn. of T.B.L.

Momentum eqn. of HBL :- $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$

(flow field)

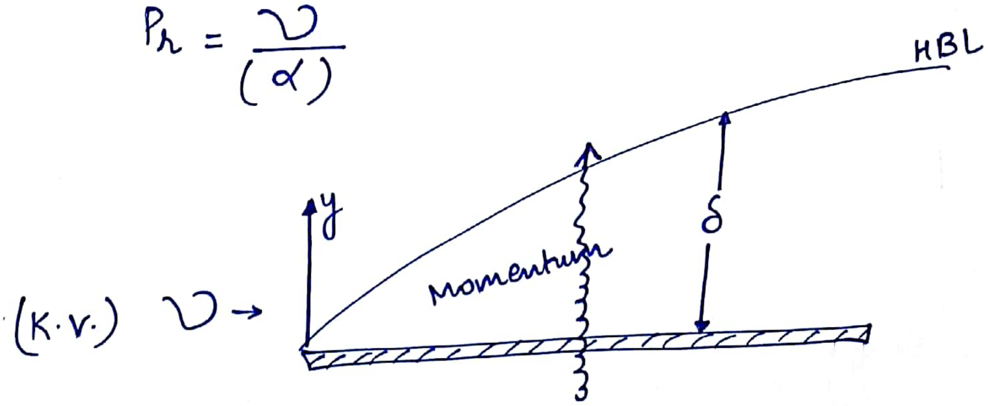
There are 2 different Equations of H.B.L :-

There is a striking similarity b/w the momentum eqn. of H.B.L. and the Energy eqn. of T.B.L.. The solution to both these second order Differential Equations would exactly be the same if K.V. (Kinematic Viscosity) ν of the fluid is equal to its thermal diffusivity 'α'.

* Significance of Prandtl No :- (Physical)

$$P_r = \frac{\nu}{(\alpha)}$$

$$\delta \propto \sqrt{\nu}$$



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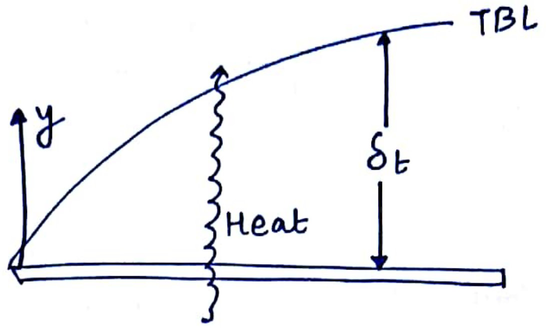
(T.D.) α →



If kinematic viscosity of the fluid is more, then viscous influence of the fluid is felt farther away into free stream thereby making the value of δ relatively more.

If the thermal diffusivity 'α' is more then the heating affects of the hot plate is felt farther away into the free stream thereby making the value of δ_t relatively more.

Thus prandtl No., a ratio b/w ν and α can tell about the relative magnitudes of HBL thickness δ and TBL thickness δ_t at a given location of 'x' measured from the leading edge of plate. → (or) same.

If $P_r > 1$	If $P_r < 1$	If $P_r \simeq 1$
$\Rightarrow \nu > \alpha$	$\Rightarrow \nu < \alpha$	$\Rightarrow \nu \simeq \alpha$
$\Rightarrow \delta > \delta_t$	$\Rightarrow \delta < \delta_t$	$\Rightarrow \delta \simeq \delta_t$
$\Rightarrow \frac{\delta_t}{\delta} < 1$	$\Rightarrow \frac{\delta_t}{\delta} > 1$	$\Rightarrow \frac{\delta_t}{\delta} \simeq 1$

for a given fluid,

$$\boxed{\frac{\delta_t}{\delta} = \frac{1}{1.026} P_r^{-1/3}}$$

P_r is a property of fluid.

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(2) d ✓

(3) b ✓

(6) $Pr = \frac{\mu C_p}{K} = \frac{0.001 \times 1000}{1}$

$\Rightarrow \delta = \delta_t = 1mm$

(22) a

(20) d

(24) for liquid Metals (Hg)

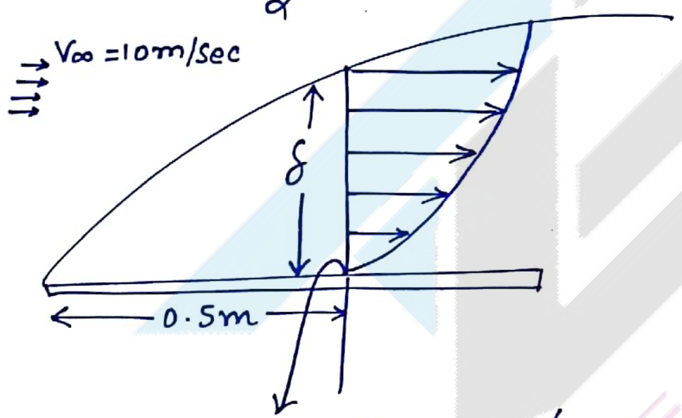
Pr is very low

$\nu <<< \alpha$

(b) ✓ $\Rightarrow \delta <<< \delta_t$

(30) $Pr = 1 = \frac{\nu}{\alpha}$

31R



$$Re_x = \frac{V_\infty x}{\nu} = \left(\frac{10 \times 0.5}{30 \times 10^{-6}}\right) = 1.6 \times 10^5$$

$$\delta_{at\ x = 0.5m} = \frac{5.0x}{\sqrt{Re_x}} = \frac{5 \times 0.5}{\sqrt{1.6 \times 10^5}} = 6.123 mm$$

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

$$(\delta_t)_{at\ x = 0.5m} = 5.96\ mm$$ Ans

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continue By $\int$ing the Energy Eqn. of T.B.L. that is

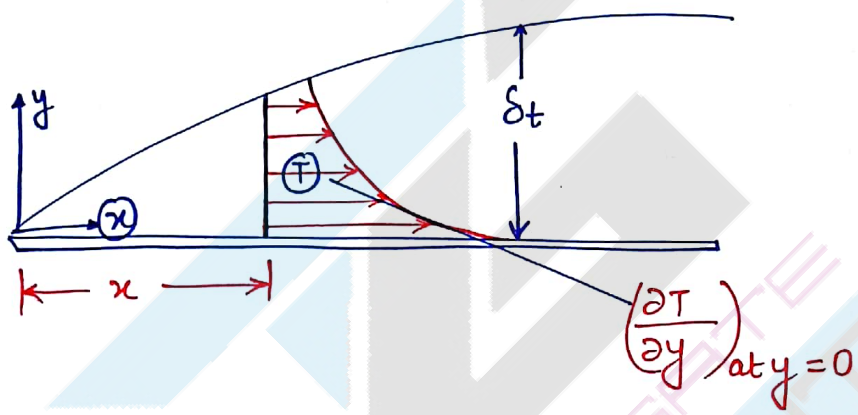
i.e. $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

with the help of its boundary conditions, we get the Tempr. distribution within the T.B.L. as :-

At any given x, measured from leading Edge of plate,

$$\boxed{\left(\frac{T - T_w}{T_\infty - T_w}\right) = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3}$$

where $\delta_t = f(x)$



hence, $$\left(\frac{\partial T}{\partial y}\right)_{at\, y=0} = (T_\infty - T_w)\frac{3}{(2 \cdot \delta_t)}$$

$$h_x = \text{local conv. H.T. coefficient} = \frac{-k_f\left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{T_w - T_\infty}$$

$$h_x = \frac{-k_f\,(T_\infty - T_w)}{(T_w - T_\infty)}\frac{3}{2\delta_t}$$

$$\boxed{h_x = \left(\frac{3k}{2\delta_t}\right) \ W/m^2K}$$

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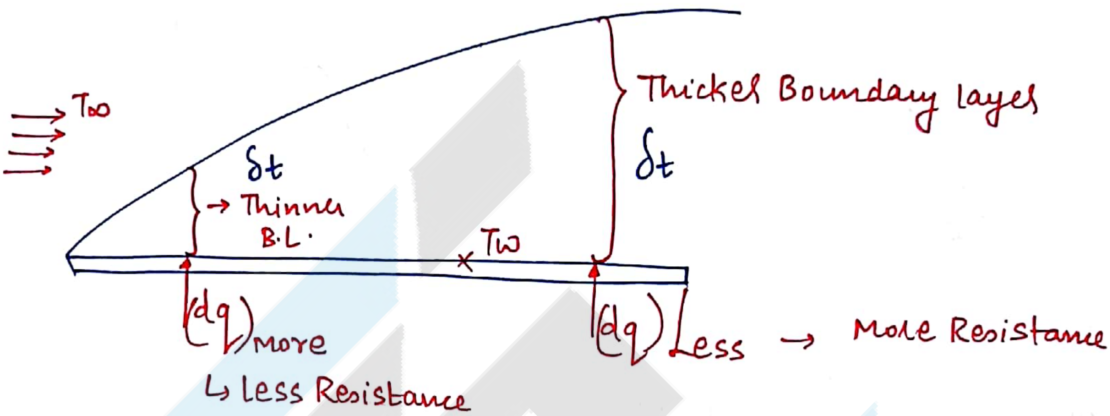
$$\delta \propto x^{1/2}$$

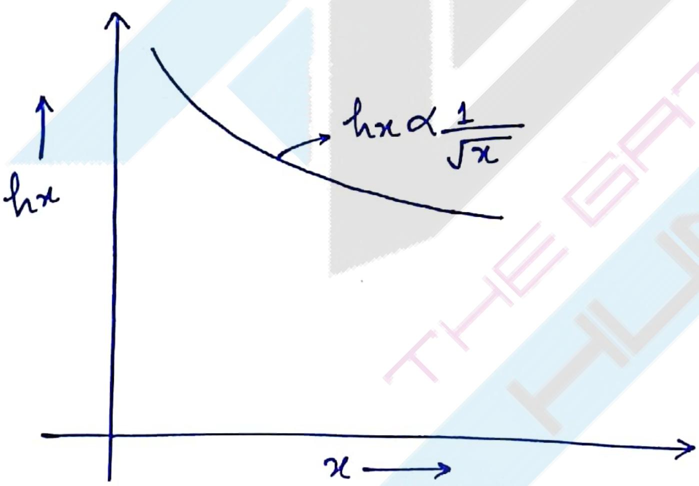
But $\dfrac{\delta_t}{\delta} \neq f(x)$ a property

Since $\dfrac{\delta_t}{\delta} = f(Pr)$

$Pr \neq f(x)$

$$\Rightarrow \boxed{\delta_t \propto x^{1/2}} \Rightarrow \boxed{h_x \propto x^{-1/2}}$$





h_x decreases with ↑ of x because the thicker Boundary layer at a greater value of x shall offer more thermal Resistance against the heat flow between the hot plate and the free stream fluid at T_∞.

$h_x \rightarrow k, \delta_t \rightarrow k, \delta, Pr \longrightarrow k, x, Re_x, Pr$

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

Let local Nusselt No. $= Nu_x = \left(\dfrac{h_x x}{k}\right)$

∴ The local convective H.T. coeff. h_x for Laminar Boundary layer over flat plate can be obtained from:-

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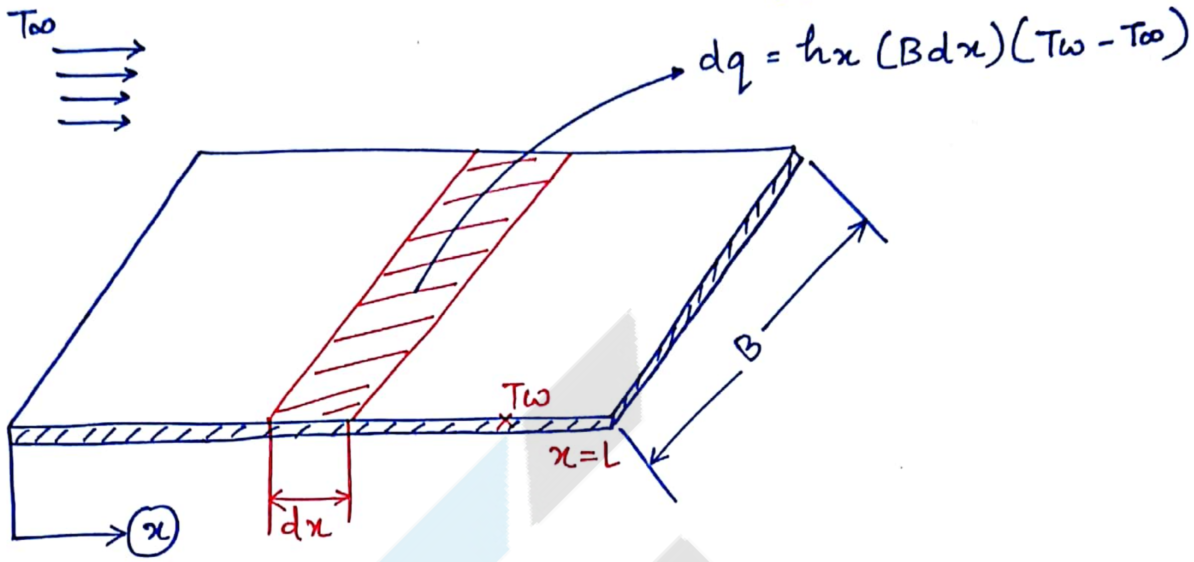
$$\boxed{Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}}$$

Local Nusselt's No.

* **AVERAGE Convective Heat Transfer coefficient ($\bar{h}$)** :→



Consider a small differential strip of the plate of length 'dx' where the differential heat transfer rate b/w the strip and fluid is dq then

$$dq = h_x B (T_w - T_\infty) dx$$

$$\int_{x=0}^{L} dq = \int_{x=0}^{L} h_x B (T_w - T_\infty) dx$$

⇒ Total H.T. Rate from entire plate = $Q = \int_{x=0}^{L} h_x B (T_w - T_\infty)\, dx \longrightarrow (1)$

But Interms of $\bar{h}$, Total H.T. Rate between entire plate and fluid = $Q = \bar{h} \times (B \times L)\, T_w - T_\infty \longrightarrow (2)$

equating (1) & (2), we get

$$\boxed{\bar{h} = \frac{1}{L} \int_{x=0}^{L} h_x\, dx}$$

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we know $h_x \propto x^{-1/2}$

$$\Rightarrow h_x = C x^{-1/2}$$

put $x = L$ on both sides

$$h_{x=L} = C L^{-1/2} \Rightarrow C = \frac{h_{x=L}}{L^{-1/2}}$$

(local convective H.T. coefficient at $x = L$ {i.e. trailing edge})

$$\bar{h} = \frac{1}{L}\int_{x=0}^{L} C x^{-1/2}\,dx = \frac{1}{L} C \left[\frac{x^{-1/2+1}}{-1/2+1}\right]_{x=0}^{L}$$

$$\Rightarrow \bar{h} = \frac{1}{L} \times \frac{h_{x=L}}{L^{-1/2}} \left[\frac{L^{1/2}}{1/2}\right]$$

$$\Rightarrow \boxed{\bar{h} = 2(h_{x=L})\ \mathrm{W/m^2K}}$$

27/9/16

Hence the average convective H.T. coefficient for the entire plate will be equal to twice the local convective H.T. coefficient at the trailing edge (i.e. $x = L$)

we know that

$$\frac{h_x x}{K} = \text{local Nu. No.} = Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

Put $x = L$ on both sides

$$\frac{h_{x=L} \times L}{K} = 0.332\, Re_L^{1/2}\, Pr^{1/3}$$

$$\Rightarrow \frac{2h_{x=L} \times L}{K} = 0.664\, Re_L^{1/2}\, Pr^{1/3}.$$

where $Re_L = \frac{V_\infty L \rho}{\mu}$

local Reynolds No. at Trailing edge.

$$\Rightarrow \boxed{\frac{\bar{h}L}{K} = \overline{Nu} = \text{Average Nu. No.} = 0.664\, Re_L^{1/2}\, Pr^{1/3}}$$

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Also $\boxed{\overline{Nu} = 2(Nu_x)_{at\ x=L}}$

Pg 85 Q(11)

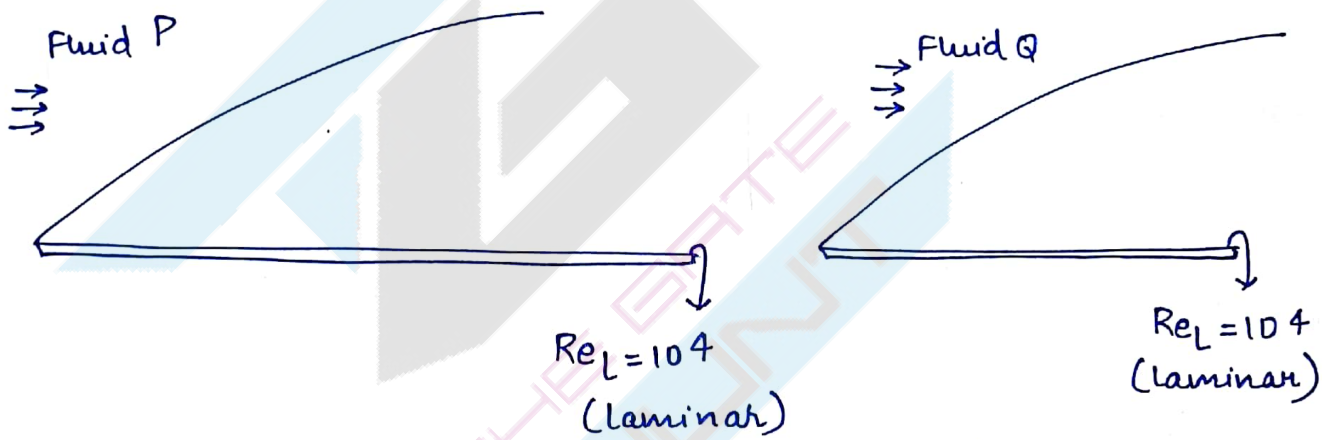
$$\left(\frac{\delta}{\delta t}\right)_P = \frac{1}{2} \qquad \left(\frac{\delta}{\delta t}\right)_Q = 2 \qquad Re_{Both} = 10^4 \qquad Pr_P = 1/8 \qquad Nu_P = 35$$

$$(Pr)_Q = ? \quad (Nu)_Q = ?$$

$$\left(\frac{\delta t}{\delta}\right) = \frac{1}{1.026} Pr^{1/3}$$

$$\frac{1}{2} = \frac{1}{1.026} Pr^{1/3}$$

SIR



$$\left(\frac{\delta t}{\delta}\right)_P = 2 \qquad \left(\frac{\delta t}{\delta}\right)_Q = \frac{1}{2}$$

$$Pr_P = 1/8 \qquad Pr_Q = ?$$

we know that,

$$\frac{\delta t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

For fluid P,

$$2 = \left(\frac{\delta t}{\delta}\right)_P = \frac{1}{1.026}(Pr_P)^{-1/3} \longrightarrow (1)$$

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For Q, $\frac{1}{2} = \left(\frac{\delta t}{\delta}\right)_Q = \frac{1}{1.026} (Pr_Q)^{-1/3} \longrightarrow (2)$

$\frac{(2)}{(1)} \Rightarrow \frac{1}{2\times 2} = \frac{(Pr_Q)^{-1/3}}{(Pr_P)}$

$\Rightarrow \boxed{Pr_Q = 8}$

$\overline{Nu}_P = 0.664\ Re_L^{1/2}\ Pr_P^{1/3} = 35 \longrightarrow (3)$

$\overline{Nu}_Q = 0.664\ Re_L^{1/2}\ Pr_Q^{1/3} = ? \ — (4)$

$\frac{(4)}{(3)} \Rightarrow \overline{Nu}_Q = 140.$

(26) $h_x = a x^{-0.1}$

$\left(\frac{\bar{h}}{h_x}\right)_x = ?$

SIR $\frac{\bar{h}}{h_x} = \frac{1/x \int_0^x h_x\, dx}{h_x} = \frac{1/x \int_0^x \cancel{a} x^{-0.1} dx}{\cancel{a} x^{-0.1}} = 1.11$

(29) $\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta t}\right) - \frac{1}{2}\left(\frac{y}{\delta t}\right)^3$ $\quad Nu_x = ?$

$\delta t \leftarrow$ TBL

SIR At any given x,

$\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = (T_\infty - T_w)\frac{3}{2\delta t}$

$\therefore h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}}{T_w - T_\infty} = \left(\frac{3k}{2\delta t}\right)$

$\therefore$ local Nu. No. = $Nu_x = \left(\frac{h_x x}{k}\right) = \frac{3\cancel{k}}{2\delta t} \times \frac{x}{\cancel{k}} = \left(\frac{3x}{2\delta t}\right)$

Local Nu. No. based on T.B.L. thickness

(Put) $x = \delta t \Rightarrow (Nu_x)_{\text{based on } \delta t} = \frac{3\times\delta t}{2\times\delta t} = 1.5.$

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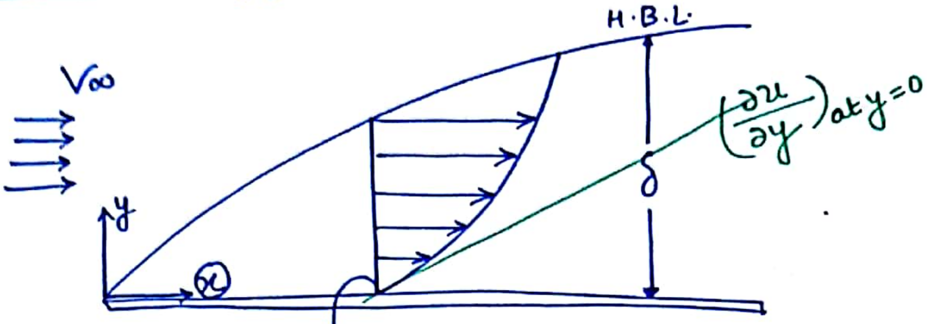
* Analogy between Fluid friction and Heat Transfer (Reynold's-Colburn analogy) :-



τ_w = local wall shear stress $= \mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0}$

$= \mu V_\infty \frac{3}{2\delta} = \mu V_\infty \frac{3}{2 \times \frac{4.64x}{\sqrt{Re_x}}}$

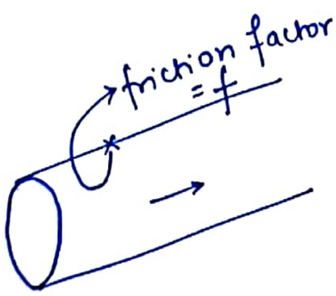
$= \mu V_\infty \times \frac{3}{2 \times 4.64x \Big/ \sqrt{\frac{V_\infty x \rho}{\mu}}}$ Pascal —— (1)

In F.M., local wall shear stress =

$\tau_{w_{all}} = C_{fx} \times \frac{\rho V_\infty^2}{2}$ Pa —— (2)

C_{fx} = local skin friction coefficient (OR) Local drag coefficient.

$= 0.004$ (OR) 0.005 {a dimensionless parameter.}



Equating (1) & (2), we get

$$\boxed{0.332\, Re_x^{-1/2} = \frac{C_{fx}}{2}}$$

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$$Nu_x = 0.332\ Re_x^{1/2} \cdot Pr^{1/3}$$

$$\Rightarrow \left(\frac{Nu_x}{Re_x Pr}\right) = \frac{0.332\ Re_x^{1/2} \cdot Pr^{1/3}}{Re_x Pr}$$

stanton No. = $\boxed{St = \frac{Nu}{Re\, Pr}} = \frac{Nu}{Pe}$ $\left[Pe \rightarrow \text{Peclet No}\right]$

$\therefore$ local stanton No. $= St_x = \dfrac{Nu_x}{Re_x Pr}$

$$\Rightarrow St_x = \frac{\frac{h_x \cancel{x}}{\cancel{k}}}{\frac{V_\infty \cancel{x} \rho}{\cancel{\mu}} \times \frac{\cancel{\mu} C_p}{\cancel{k}}}$$

$$\Rightarrow \boxed{St_x = \frac{h_x}{(V_\infty \rho C_p)} = \frac{1}{\eta_2}}$$

$$\Rightarrow \boxed{St_x = \frac{1}{\eta_2}}$$

The product of $(Re\ Pr)$ is called as Peclet No. (Pe). significant in liquid Metal cooling of Nuclear Reactors.

$$\therefore \boxed{St_x = \left(\frac{Nu}{Pe}\right)}$$

also, $St_x = \left(0.332\ Re_x^{-1/2}\right) Pr^{-2/3}$

$$\boxed{St_x\ Pr^{2/3} = \frac{C_{fx}}{2}} \rightarrow \text{Reynold's Analogy}$$

* PHYSICAL Significance of Reynold's Analogy :-

from this Reynolds analogy, we can predict the value of local convective H.T. coefficient h_x at a given location of x measured from the leading edge just by knowing the local skin friction coefficient C_{fx} at the same location of x even when there is no Heat Transfer b/w the plate and the flowing fluid.

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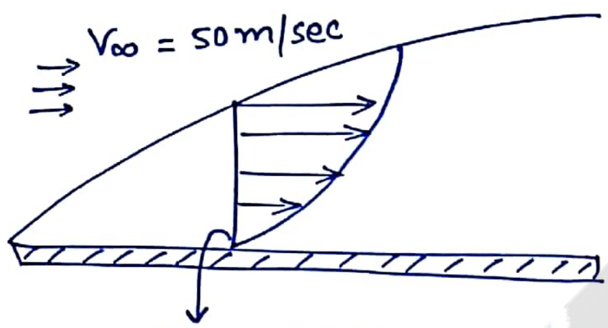
Pg 87
(31) $V_\infty = 50\, m/s$

$C_{fx} = 0.004$

$h_x = ?$

$$St_x = \frac{h_x}{\rho V_\infty C_p}$$

SIR



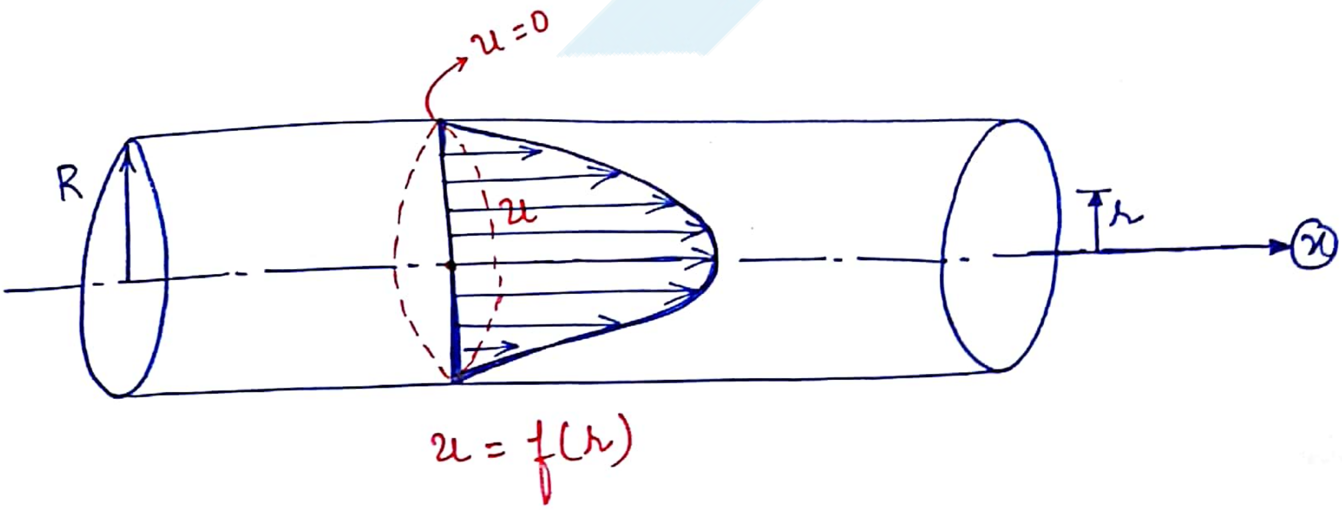
$C_{fx} = 0.004$

$$Pr = \frac{\mu C_p}{k} \quad (C_p \rightarrow J/kg\,K) = 0.653$$

$$St_x\, Pr^{2/3} = \frac{C_{fx}}{2} = \frac{0.004}{2}$$

$\Rightarrow St_x =$ local stanton No. $= 2.65 \times 10^{-3} = \dfrac{h_x}{\rho V_\infty C_p}$ ($C_p \rightarrow J/kg\,K$) $\Rightarrow h_x = 116.9\ W/m^2$

FORCED CONVECTION in Flow through Pipes/Ducts :-



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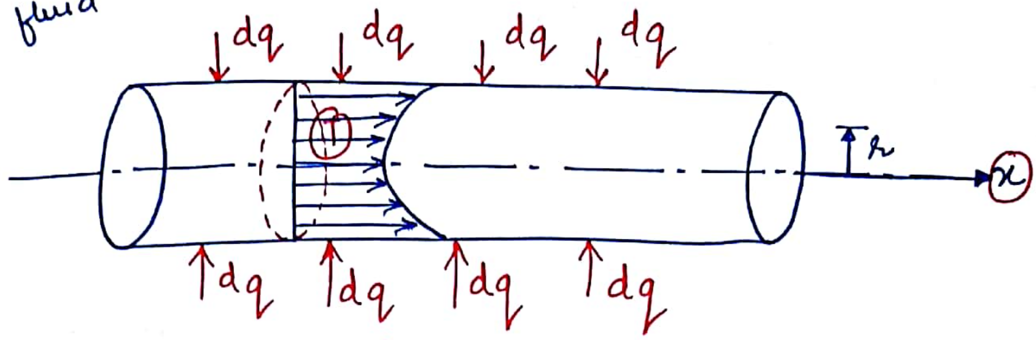
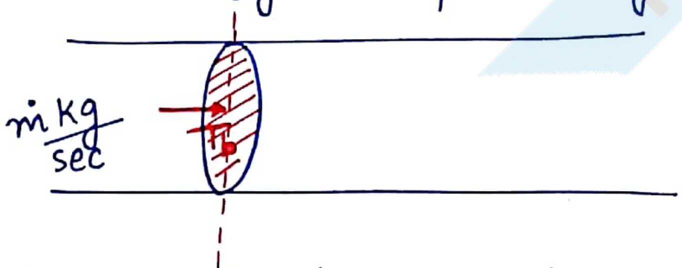
Mass flow rate of flowing fluid $\boxed{\dot{m} = \rho \times \pi R^2 V_{mean}\ kg/sec}$



$T = f(r)$

Just like velocity of fluid layers being a function of 'r' (measured from the axis) at a given cross-section of the pipe whenever there is fluid flow happening in the pipe due to viscous influence of the fluid, whenever there is H.T. b/w the pipe boundary and the flowing fluid, the tempr. of fluid layers also become a function of r at a given cross-section.

$\boxed{T_b \text{ (Bulk Mean Tempr. of Fluid)}}$:- T_b of fluid at a given c/s of pipe is defined as the temperature which takes into account the variation of tempr. of fluid layers with respect to 'r' at that c/s of the pipe and thus indicates the total thermal Energy transported by the fluid through the c/s.



$\therefore$ The thermal energy/enthalpy transported by fluid through the c/s of the pipe $= \dot{m} C_p T_b = \rho \pi R^2 V_{mean} C_p T_b\ J/sec$

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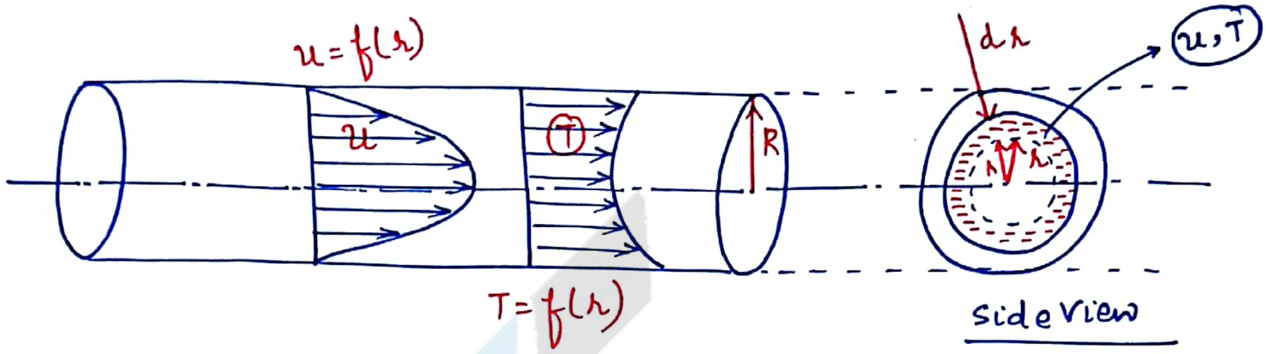
If there is any kind of H.T. b/w the pipe boundary and the flowing fluid. This T_b of fluid must change in the direction of fluid flow.

* **TO DERIVE an expression for T_b of fluid :-**



Consider a given c/s of the pipe at which the velocity distribution and the tempr. distribution are as shown in the figure. Consider a differential elemental ring of fluid flow in the c/s of the pipe at a radius of 'r' measured from the axis. Let 'dr' be the differential radius of the elemental ring then

$d\dot{m}$ = Differential mass flow Rate of fluid through the elemental Ring $= \rho \times 2\pi r\, dr\, u$.

∴ Differential Thermal energy (or) enthalpy transported by fluid through elemental Ring $= d\dot{m}\, C_p T$ J/sec

$= \rho\, 2\pi r\, dr\, u\, C_p T$ J/sec

∴ Total Thermal Energy transported by fluid through entire cross-section of pipe $= \int_0^R \rho\, 2\pi r\, u\, C_p T\, dr$. → (1)

But Interms of T_b,

Total Thermal Energy transported by fluid through entire cross-section $= \rho \pi R^2 V_{mean} C_p T_b$. J/sec → (2)

Equating (1) & (2),

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we get $\cancel{\rho}\cancel{\pi}R^2 V_{mean}\cdot\cancel{c_p}\cdot T_b = \int_0^R \cancel{\rho}\,2\cancel{\pi} r u \cancel{c_p} T\,dr$

$$R^2 V_{mean} T_b = \int_0^R 2 u r T\,dr$$

$$\Rightarrow \boxed{T_b = \frac{2\int_0^R r u T\,dr}{R^2 V_{mean}}}$$

21 (Pg 86) $u(r,x) = C_1$

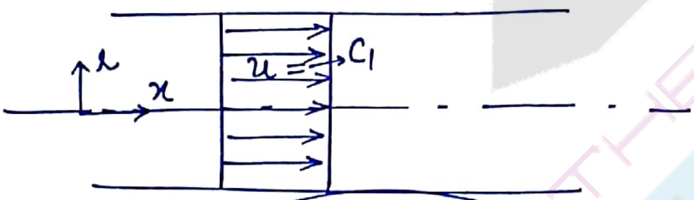
$$T(r,x) = C_2\left[1-\left(\frac{r}{R}\right)^3\right]$$

$$T_m = \frac{2}{U_m R^2}\int_0^R u(r,x)\,T(r,x)\,r\,dr$$

$$T_m = \frac{2}{U_m R^2}\int_0^R C_1 C_2\left(1-\left(\frac{r}{R}\right)^3\right) r\,dr$$

$T_m =$

S/R $u(r,x) = C_1$

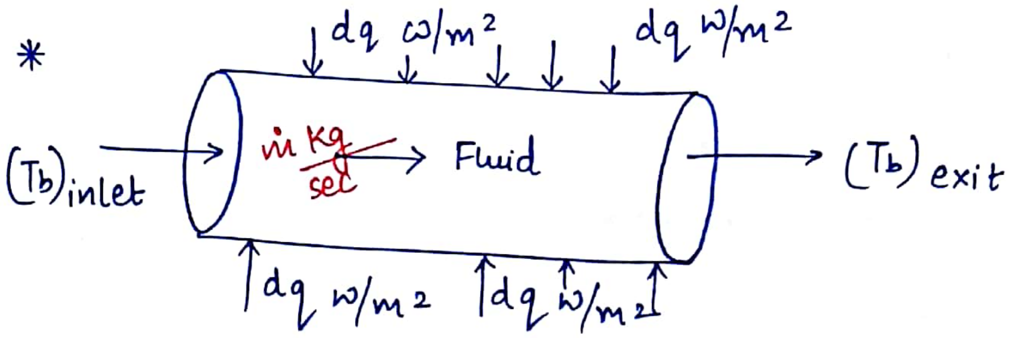


$V_{mean} = C_1 = V_m$

$$\therefore T_b = \frac{2}{\cancel{U_m}R^2}\int_0^R \cancel{C_1}\,C_2\left[r-\frac{r^4}{R^3}\right]dr$$

$$T_b = 0.6C_2$$

*



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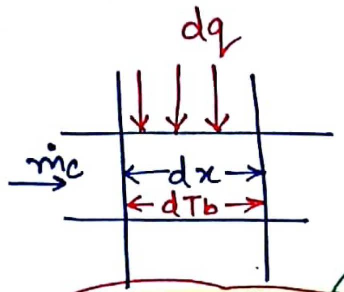
Total H.T. Rate b/w entire Pipe and flowing fluid

$$= \dot{m} c_p (\Delta T)_b = \dot{m} c_p (T_{b\,exit} - T_{b\,inlet}) \text{ watt}$$



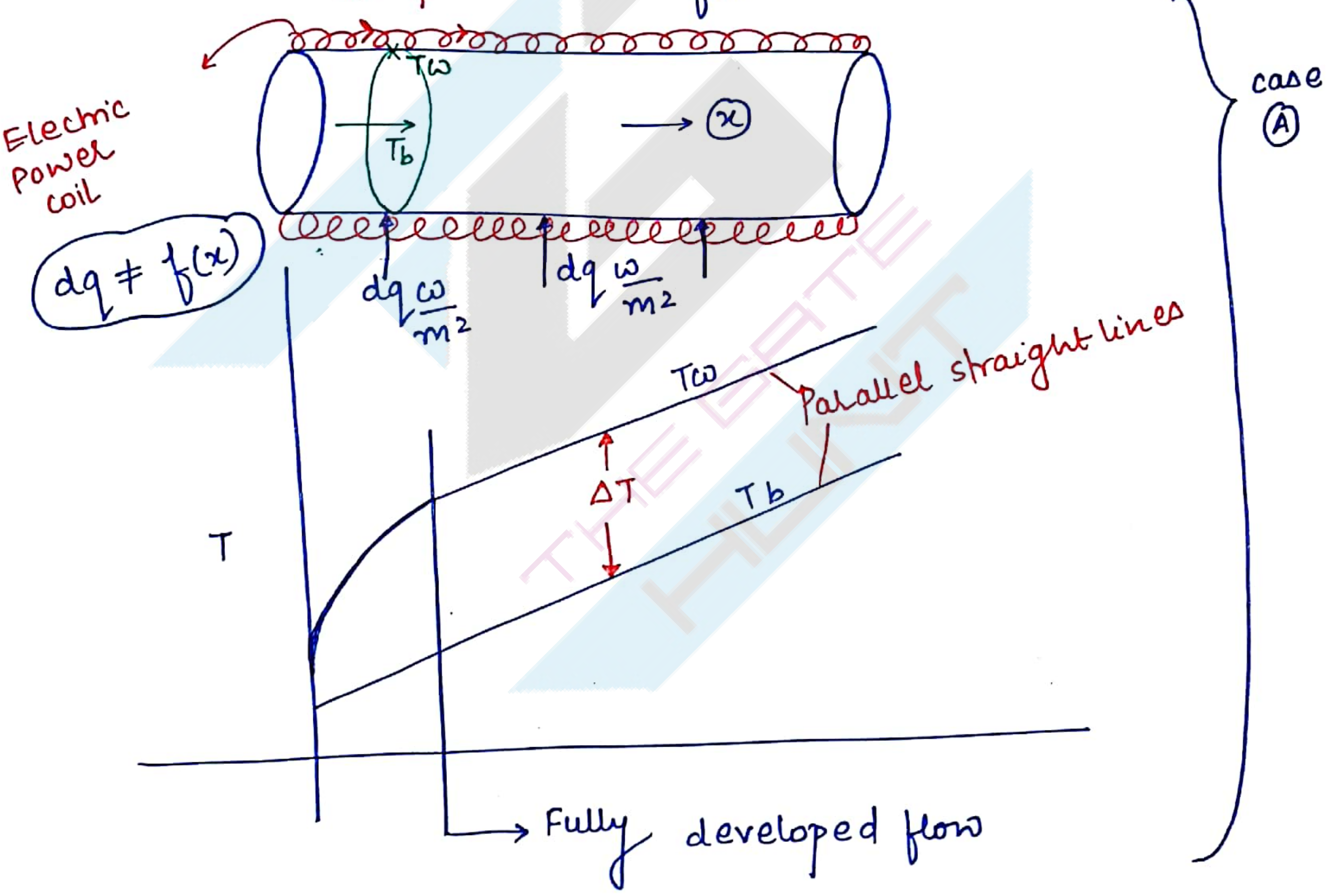
$$dq = \dot{m}_c \, c_p \, dT_b$$

enthalpy changes indicated by this Term.

differential form

* $\boxed{\text{VARIATION OF } T_b}$:-

i Amp constant Heat flux conditions.



$$dq \neq f(x)$$

Newton's law of cooling at any given x :-

$$dq = \text{local heat flux} = h_x \times 1 \times (T_w - T_b) \, (W/m^2)$$

$$dq = h_x \times 1 \times (\Delta T)$$

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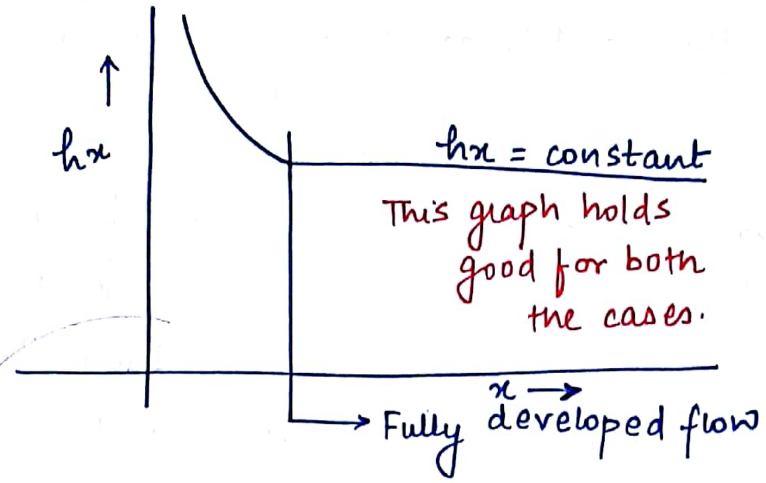
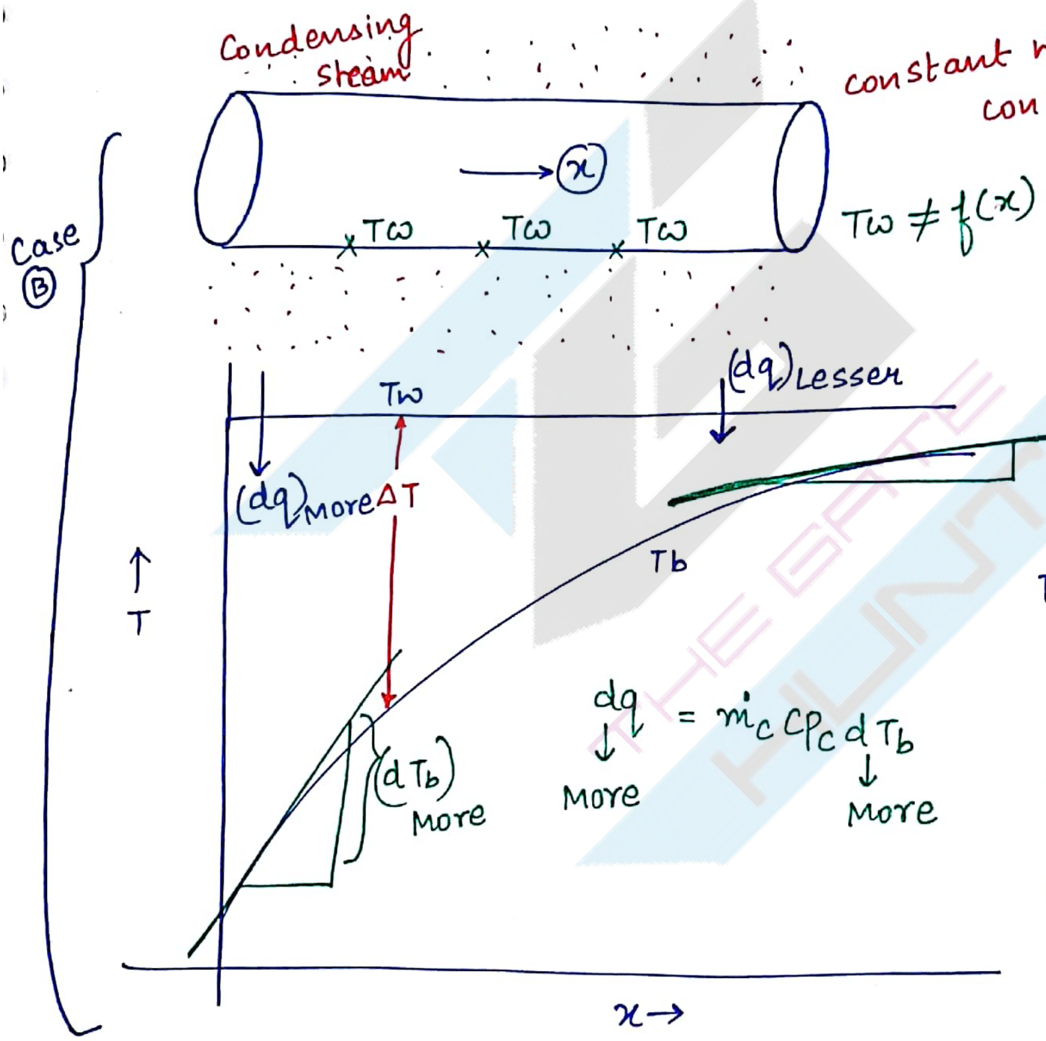


constant wall Temp. conditions



Tw = Pipe wall Temperature

Tb = Bulk Mean Temp. of fluid

dq = Local Heat flux

$x \rightarrow$ (indicates the direction of flow)

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Unlike in case of flow over flat plates, the local convective H.T. coefficient h_x remains constant in the direction of fluid flow during both constant heat flux conditions as well as constant wall tempr. conditions. ⟶ After case Ⓐ ⟶ T_b should ↑ in the dirn. of fluid i.e. during constant heat flux conditions, since both dq and h_x are remaining constant in the dirn. of fluid flow, the ~~local~~ ΔT value also must remain constant in the dirn. of fluid flow. But since T_b has to ↑ in the dirn. of fluid flow (Because the fluid is getting heat), T_w value also must increase in the direction of fluid flow in such a way that $T_w - T_b$ shall remain the same at any x.

After case Ⓑ → During constant wall tempr. conditions, since T_w is remaining constant and T_b has to increase in the dirn. of fluid flow, the ΔT value must be decreasing in the dirn. of fluid flow.

Since h_x is remaining constant & ΔT is ↓ing in the dirn. of fluid flow, the local heat flux dq must decrease in the dirn. of fluid flow. This is evident from the decreasing slope of $\left(\frac{dT_b}{dx}\right)$ with respect to x.

✓ When dq = constant $\neq f(x)$

⇒ T_w is increasing with x

and

✓ when T_w = constant

⇒ dq is decreasing with x.

Hence, it is just not possible to maintain both constant heat flux conditions and constant wall tempr. conditions simultaneously at the same time.

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h_x = constant

Local Nusselt No. → Diameter

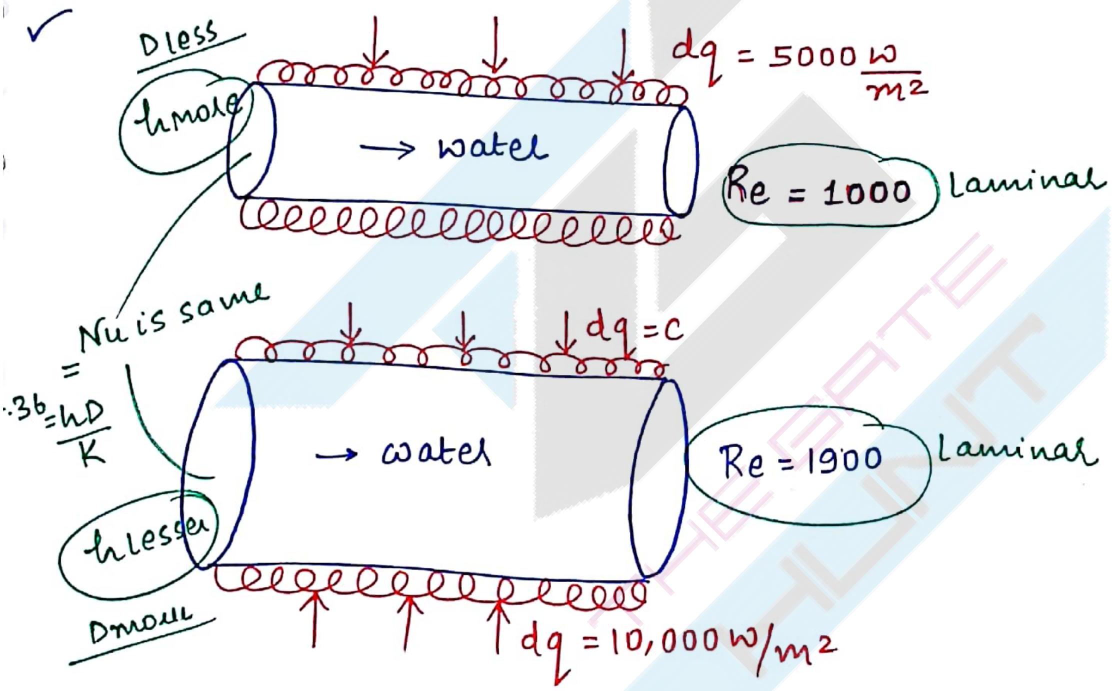
$$Nu_x = \frac{h_x D}{K} = \text{constant}$$

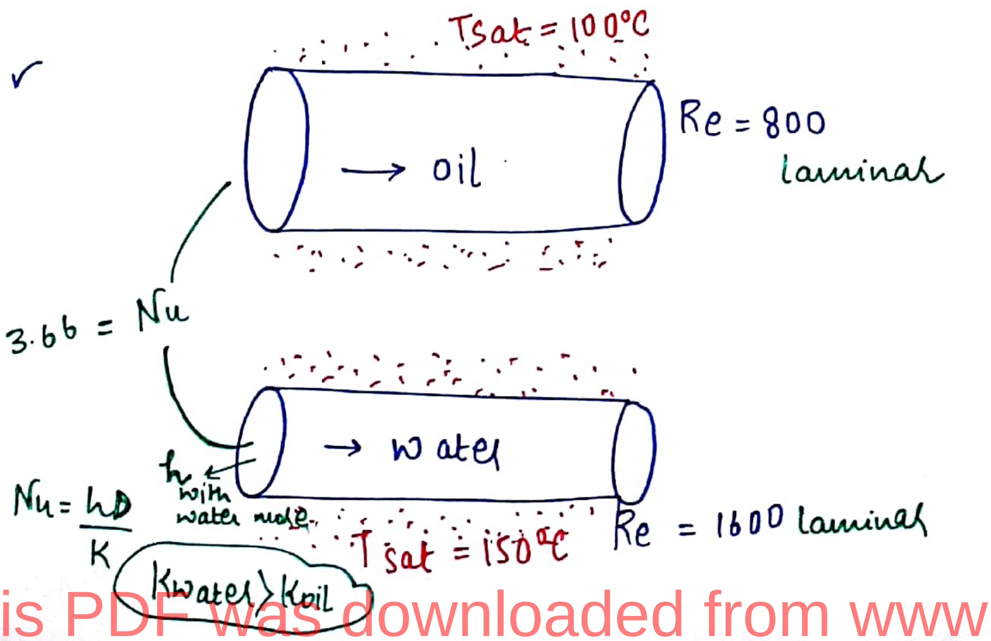
(During both constant heat flux and constant wall tempr. conditions).

∴ For fully developed laminar flow through (Re < 2000) Pipes/Ducts,

$$\boxed{Nu = \frac{hD}{K} = 4.36}$$ (During constant Heat flux conditions)

$$\boxed{Nu = \frac{hD}{K} = 3.66}$$ (During constant wall Tempr. conditions)





→ if Velocity doubles, then

$$Re = \frac{\rho V d}{K}$$ → D

Re = 1600

No change in Nu No.

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For fully developed Turbulent flow through pipes or Ducts :-

'h' can be obtained from :-

$$\boxed{Nu = \frac{hD}{K} = 0.023\, Re^{0.8}\, Pr^{n}}$$

Mcadam's equation (OR) Dittus- Boelter Equation

$n = 0.4$ for heating of fluid.

$n = 0.3$ for cooling of fluid.

Pg 87

(28) $D = 25mm$

$V_{mean} = 1.0\ m/s$

$\rho = 1000\ kg/m^3$

$\mu = 7.25 \times 10^{-4}\ N s/m^2$

$K = 0.625\ W/m.K$

$Pr = 4.85$

$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$

$\cancel{\frac{\rho V D}{K}} = 40 = Re$

SIB

$$Nu = \frac{hD}{K} = 0.023 \left(\frac{VD\rho}{\mu}\right)^{0.8} \left(\frac{\mu c_p}{k}\right)^{0.4}$$

$D = 0.025m$

$v = 1m/sec$

$h = 4,613.6\ W/m^2K$ ↑high.

Pg 28

(7) $1m \times 0.5m$ $Nu = 0.023\, Re^{0.8}\, Pr^{0.33}$

$T_\infty^{\omega} = 20°C$

$V_\infty = 10m/s$

$T_\infty = 30°C$

$K = 0.025$

$\mu = 18\ \mu Pas$

$Pr = 0.73$

$\rho = 1.2$

$Nu = 3.4$

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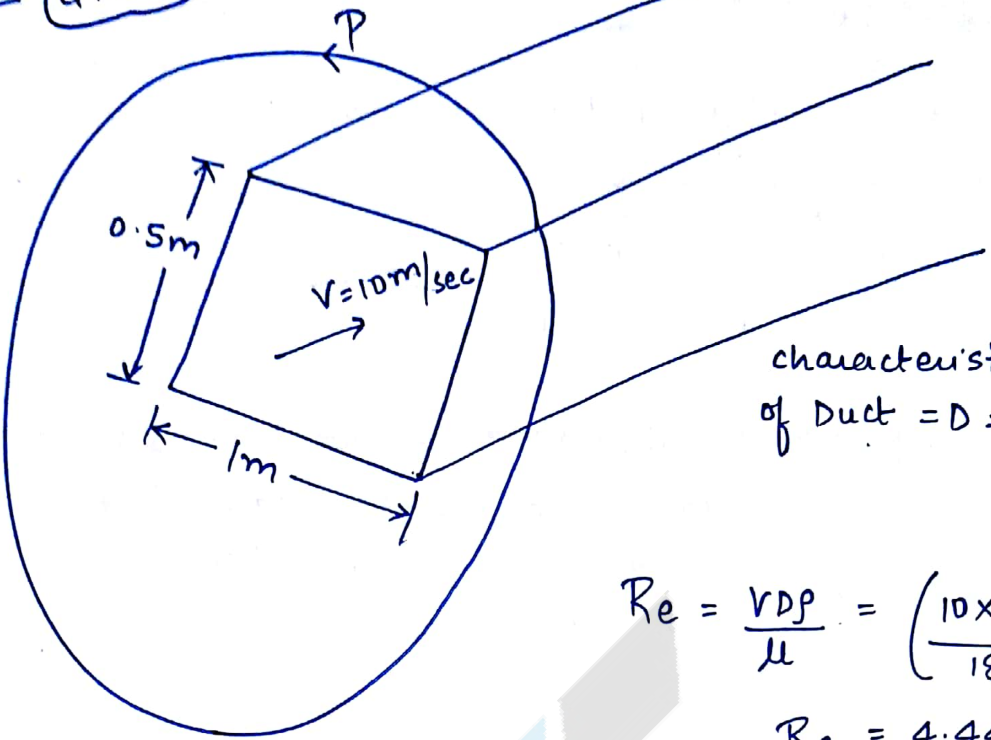
SIR (Q7 & Q8)



characteristic dimension of Duct $= D = \dfrac{4A_{c/s}}{P} = \dfrac{4\times(1\times0.5)}{2(1+0.5)}$

$= 0.667\,m$

$$Re = \frac{VD\rho}{\mu} = \left(\frac{10\times0.667\times1.2}{18\times10^{-6}}\right)$$

$$Re = 4.44\times10^5$$

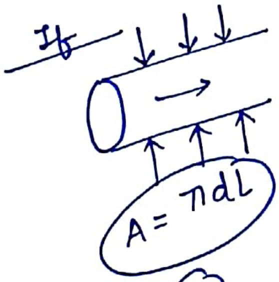
Since $Re > 4000 \Rightarrow$ Flow is turbulent (Internal flow)

$$Nu = \frac{hD}{K} = 0.023\,Re^{0.8}\,Pr^{0.33} \Rightarrow h = 25.6\,W/m^2K$$

$\therefore$ H.T. Rate per unit length of Duct $= h\times(P\times L)(T_\infty - T_\infty)$

$= 25.6\times2(1+0.5)\times1(30-20)$

$= 769$ watt.



(18) and (19)

$C_p = 4.18\times10^3\ J/kgK$

$\dot{m} = 0.01\ kg/s$

$T_w = 20°C$

$D = 50\,mm$

$L = 3\,m$

$q_w \rightarrow W/m^2$

$q_w = 2500x$

$T_{mean} = ?\ T_b = ?$

$dq = \dot{m}c_p\,dT_b$

$2500 =$

$179.42 + 20 =$

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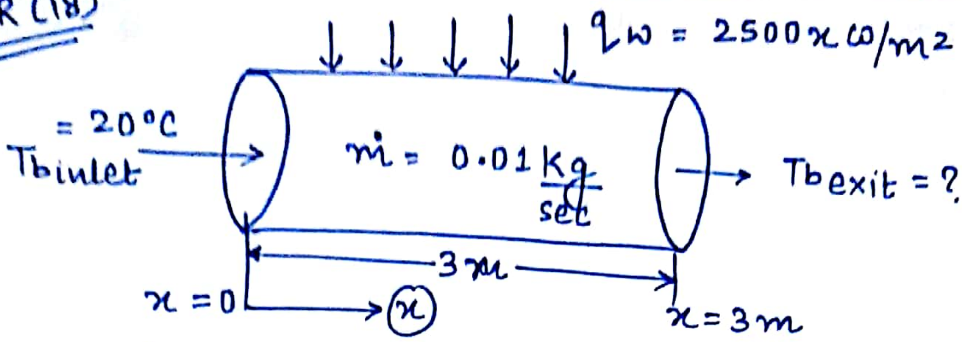
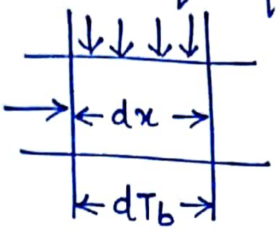
SIR (18)



$dq = q_w \times \text{Area} = q_w \times \pi D \times dx$



Differential H.T. Rate through differential H.T. area of length $dx = dq = \dot{m} C_p dT_b$

$$q_w \times \pi D dx = \dot{m} C_p dT_b$$

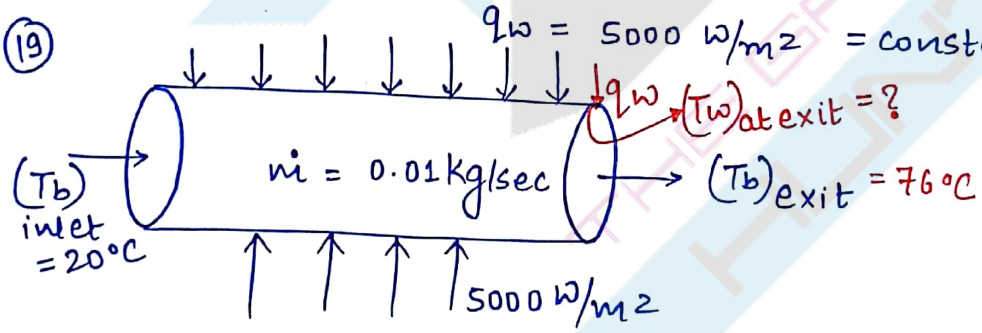
$$\int_{x=0}^{3m} 2500x \times \pi D dx = \int_{(T_b)inlet}^{(T_b)exit} \dot{m} C_p dT_b$$

$$\Rightarrow 2500 \times \frac{3^2}{2} \times \pi \times \frac{50}{1000} = 0.01 \times 4180 (T_{b\,exit} - 20)$$

Total H.T. Rate for entire pipe

$$\Rightarrow \boxed{(T_b)_{exit} = 62°C}$$

(19)



Total H.T. Rate between entire pipe and water = Heat flux × Total H.T. area

$$= 5000 \times (\pi D \times L) = \dot{m} C_p (T_{b\,exit} - T_{b\,inlet})$$

$$= 0.01 \times 4180.0 (T_{b\,exit} - 20)$$

$$\Rightarrow (T_{b\,exit}) = 76°C$$

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Newton's law of cooling at exit :-

q_w = Heat flux at exit = $h_x \times 1 (T_{w\,at\,exit} - T_{b\,at\,exit})$

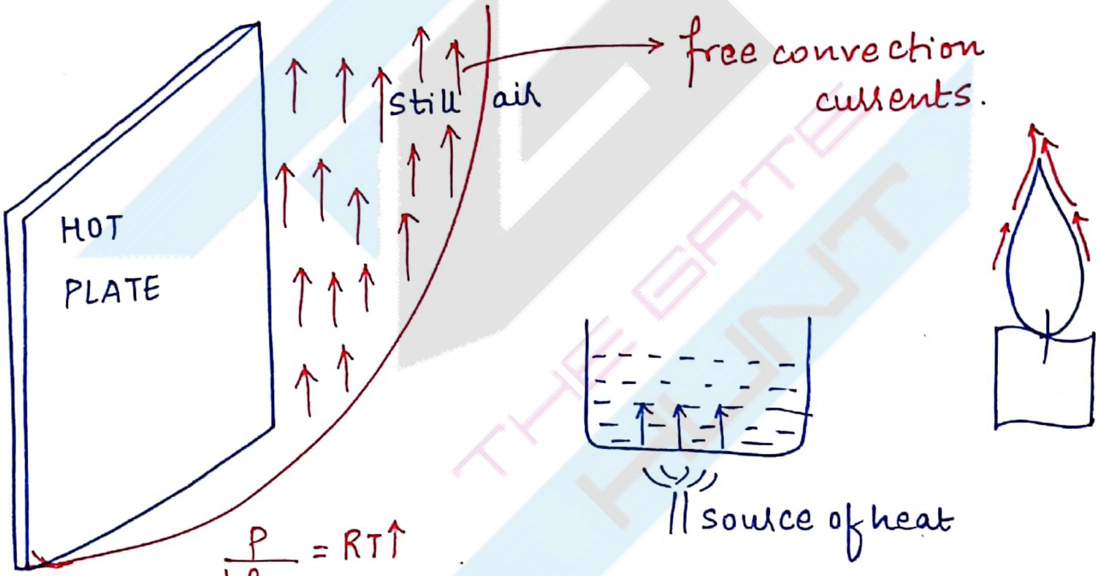
$5000 = 1000 \times 1 (T_{w\,at\,exit} - 76)$ W/m2

$\boxed{T_{w\,at\,exit} = 81^\circ C}$

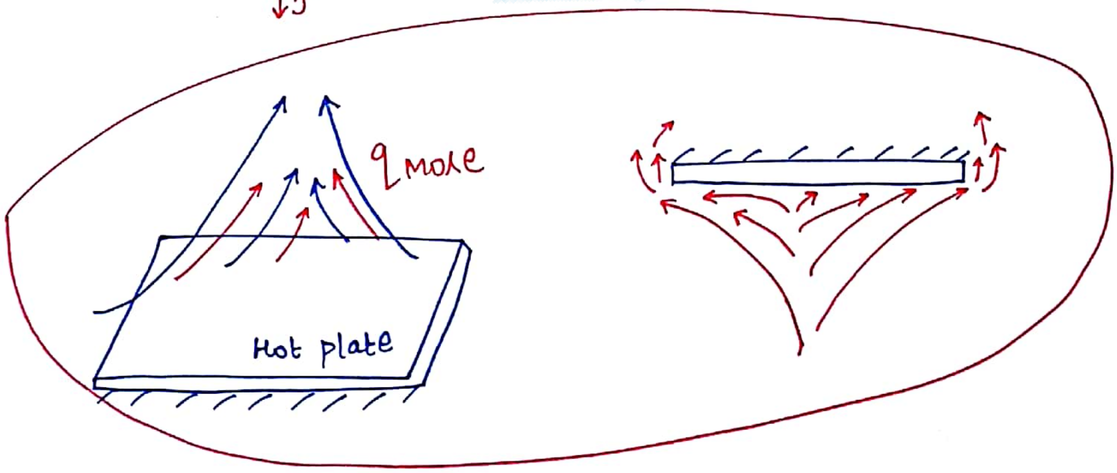
Throughout the subject, isobaric Heat transfer only.

FREE / CONVECTION
NATURAL

No velocity evident but the flow occurs naturally due to Buoyancy forces arising out of density changes of fluid.



$\frac{P}{\downarrow \rho} = RT\uparrow$



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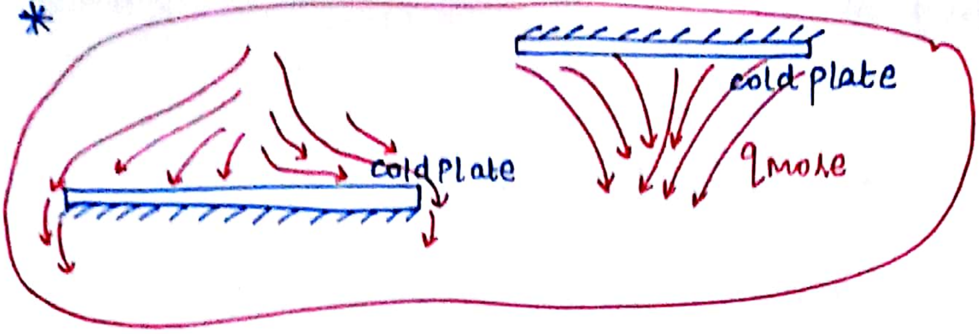

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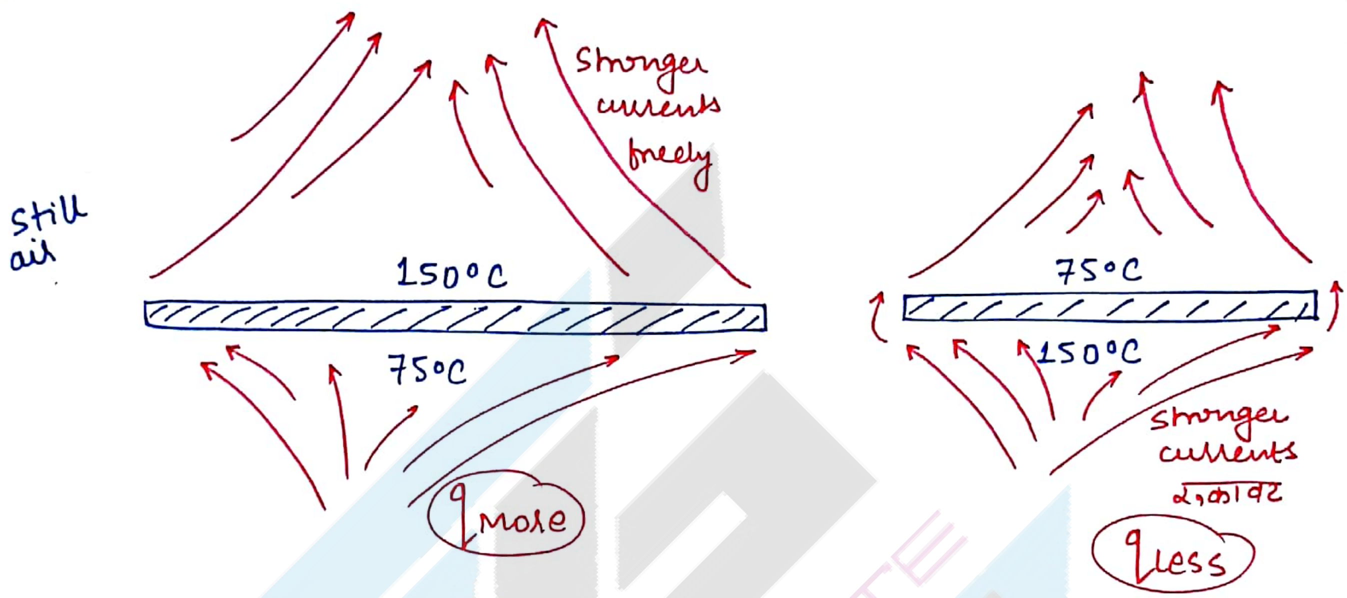

In any free convection heat transfer,

$$h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, C_p, k}_{\text{Thermophysical Properties of fluid.}})$$

g = Acceleration due to gravity.

β = isobaric Volume expansion coefficient of fluid.

$$= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P / \text{Kelvin}.$$

For ideal gas like air,

$$\beta = \frac{1}{T_{mean}} / K,$$

where T_{mean} = Mean film Temp. of fluid in K

$$= \frac{1}{\left(\frac{T_w + T_\infty}{2}\right) \text{in } K}.$$

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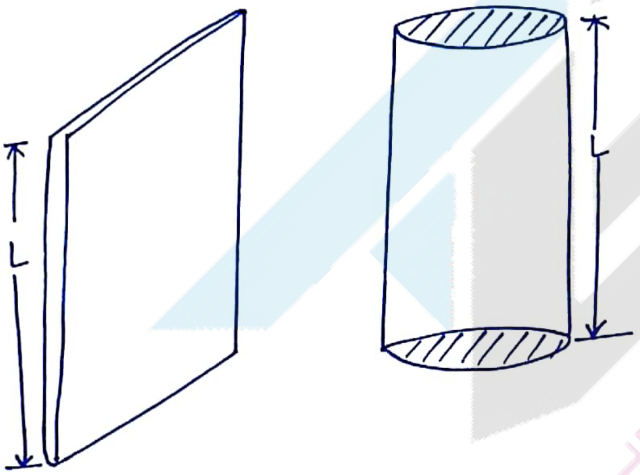
High-β ⇒ More ΔV
Gen
⇒ Higher $\Delta \rho$
⇒ Stronger Buoyancy forces

$\beta_{air} > \beta_{water}$

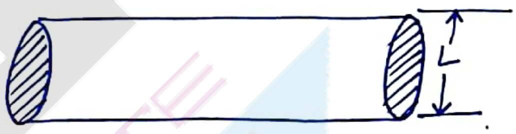
- $\Delta T = (T_w - T_\infty)$
  - T_w → Body
  - T_∞ → fluid.

- L = characteristic Dimension of Body (Dimension of Body i.e. used in the calculation of Dimensionless No.s)

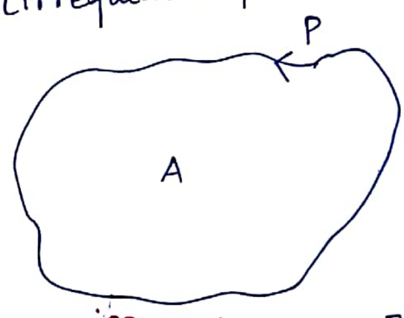
For vertical plates and cylinders,



For Horizontal cylinder,

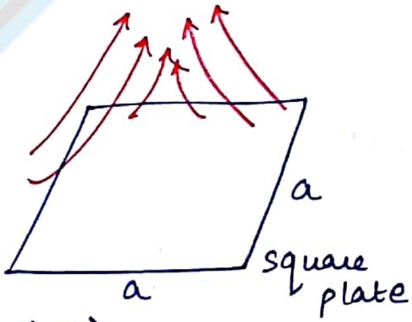


For Horizontal plate, (irregular shape)



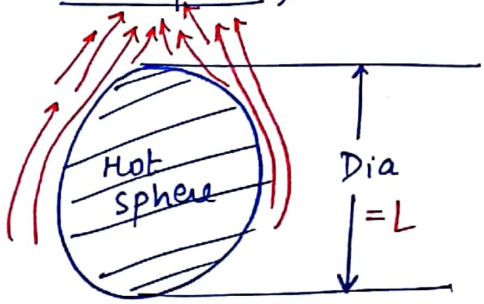
$$\boxed{L = \frac{A}{P}}$$

For ex:-



$$L = \left(\frac{a^2}{4a}\right)$$

For sphere,



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All the variables in free convection H.T. are grouped into 3 Dimensionless numbers from dimensional analysis which are given as

① $\boxed{\text{Grashoff No}}$ = $\boxed{Gr = \frac{g\beta \Delta T L^3}{\nu^2}}$ → only for gas

$= \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{viscous Force})^2}$

where ν = K.V. of fluid.

Gr signifies the Magnitude of Buoyancy forces since it contains β.

Gr replaces Reynold's No ($\because$ No velocity) in Free convection heat transfer.

② $\boxed{\text{Nusselt's No.}}$ $\boxed{Nu = \frac{hL}{K}}$

③ $\boxed{\text{Prandtl No}}$. $\boxed{Pr = \left(\frac{\mu C_p}{K}\right)}$

$\therefore$ In any free convection H.T.,

$$\boxed{Nu = f(\underbrace{Gr, Pr}_{\text{common powers}})}$$

In forced convection H.T.,

$$Nu = f(\underbrace{Re, Pr}_{\text{different exponents.}})$$

The product of Gr Pr is called

Rayleigh No. (Ra)

Usually the functional Relationship appears as :-

$$\boxed{\frac{hL}{K} = Nu = C(Gr\,Pr)^m}$$

C and m are constants which vary from case to case.

$m = 1/4$ for Laminar flow.

$m = 1/3$ for Turbulent flow.

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The flow in free convection H.T. is decided as laminar or Turbulent based on the value of $(Gr\,Pr)$ product that is Rayleigh No. (Ra) (211)

If $Gr\,Pr < 10^9 \Rightarrow$ Flow is laminar.
i.e. (Ra)

If $Gr\,Pr > 10^9 \Rightarrow$ Flow is Turbulent.

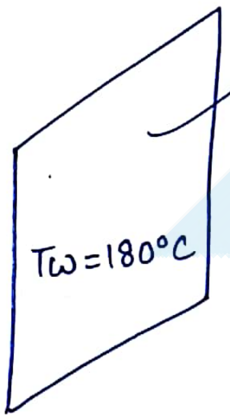
WB
(15) Pg 85

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{180-20\ g\beta L^3}{\nu^2}$$

$$Gr = 160$$

$$Gr =$$

SIR

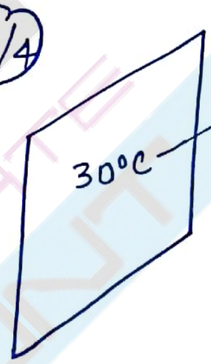


q free conv. (laminar flow)

$T_\infty = 20°C \Rightarrow m = 1/4$

$\Delta T_1 = 160°C$

$Nu_1 = 48$



$T_\infty = 20°C$

free conv

$\Delta T_2 = 10°C$

$Nu_2 = ?$

$$Nu \propto (Gr)^{1/4}$$

$$Nu \propto (\Delta T)^{1/4}$$

$$\frac{Nu_2}{Nu_1} = \left(\frac{\Delta T_2}{\Delta T_1}\right)^{1/4} \Rightarrow Nu_2 = 48 \times \left(\frac{10}{160}\right)^{1/4} = 24$$

(17) $T_\infty = 25°C$ $\quad c_p = 2.5\times10^3\ J/kg\,K$

$A = 0.01\,m^2$ $\quad T_w = 225°C$ $\quad \frac{dT}{dt} = -0.02\ K/s$

$m = 4\,kg$

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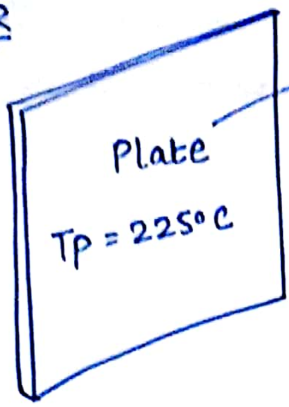
(17) SIR



Free convection

$T_\infty = 25°C$

The Rate of convection H.T. b/w plate and fluid = The Rate of decease of I.E. of plate wrt time

writting the Enegy Balance for the plate,

$$hA(T_p - T_\infty) = -mC_p\left(\frac{dT}{dt}\right) \text{ J/sec}$$

C_p: J/kg-K

$$h \times 0.1 \times (225 - 25) = 4 \times 2500 \times 0.02 \text{ J/sec}$$

$$h = 10 \text{ W/m}^2\text{K}$$

(16) $Nu = \frac{hD}{K} = 25$, D = 0.1 m

$h = 7.5 \text{ W/m}^2\text{K}$

(25) a $m = 1/4$

(12) Re = 1500

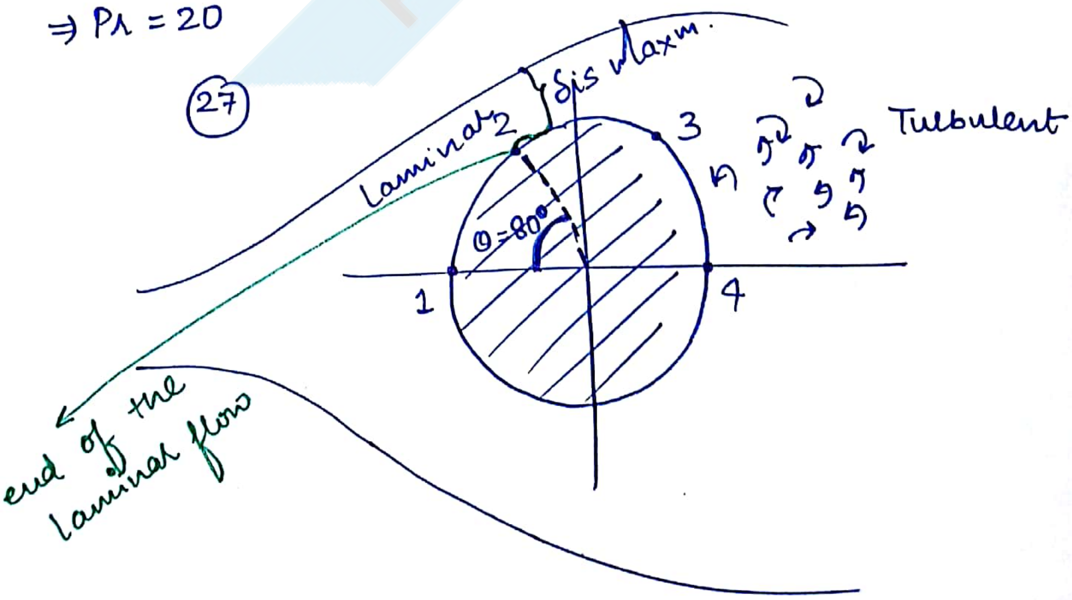
flow is Laminar

$\frac{hD}{K} = Nu = 4.36 \Rightarrow h = 43.6 \text{ W/m}^2\text{K}$ (during constant heat flux conditions.)

$\frac{hD}{K} = 3.66 \Rightarrow h = 36.6 \text{ W/m}^2\text{K}$ (During constant wall Temp. conditions.)

(10) $St = \frac{Nu}{Re\,Pr} \Rightarrow Pr = 20$

(27)



(13) a

(14) d

(23) c

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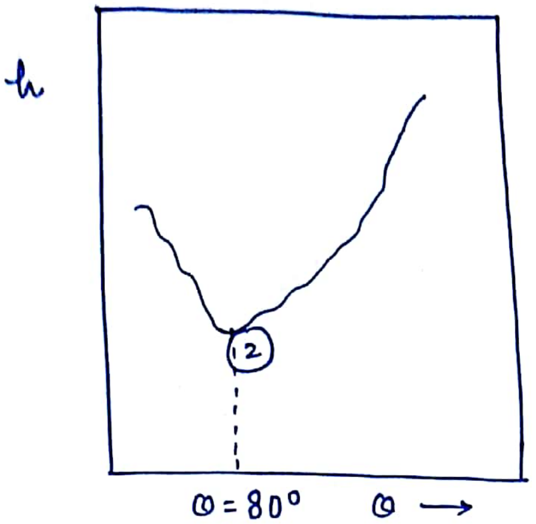

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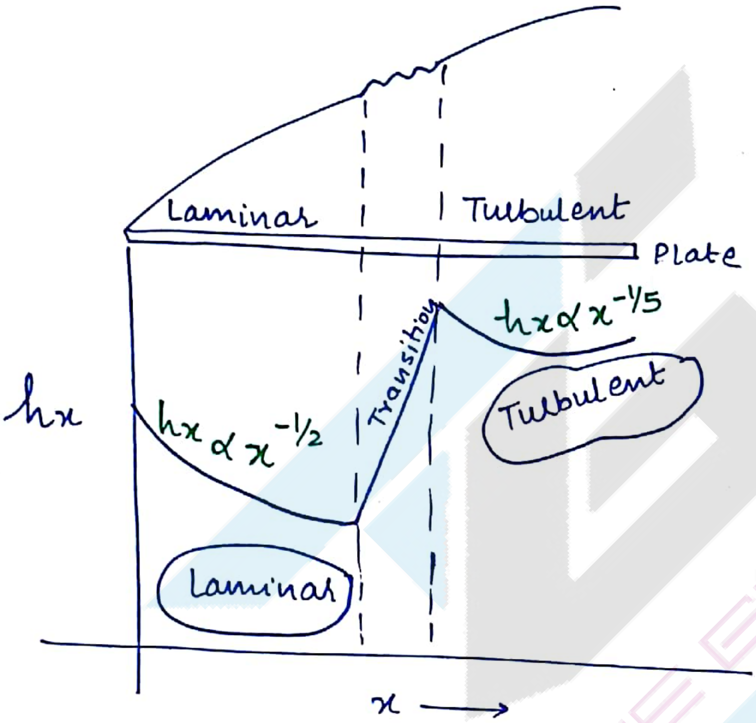

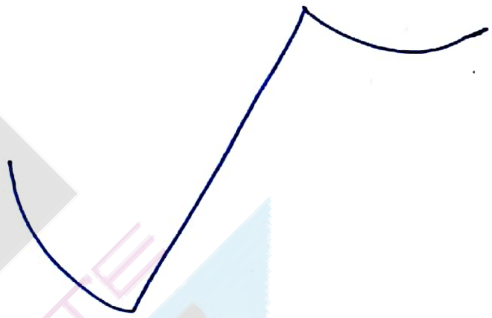

* POOL BOILING CURVE :-

The Boiling of any liquid can begin only when the liquid comes into contact with a solid surface whose tempr. is greater than the saturation temperature ~~corresponding~~ of the liquid corresponding to its saturation pressure.

Ex :- At a $p_{sat} = 101.3 \text{ kPa (abs)}$

$\Rightarrow (T_{sat})$ of water $= 100°C$.

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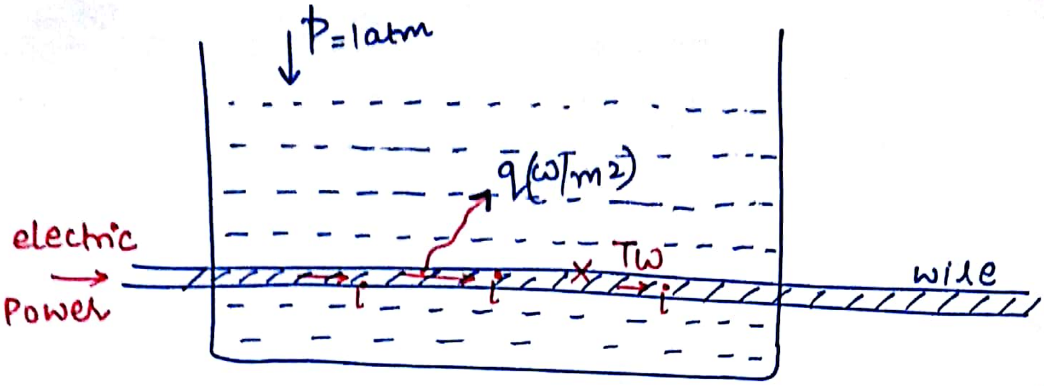

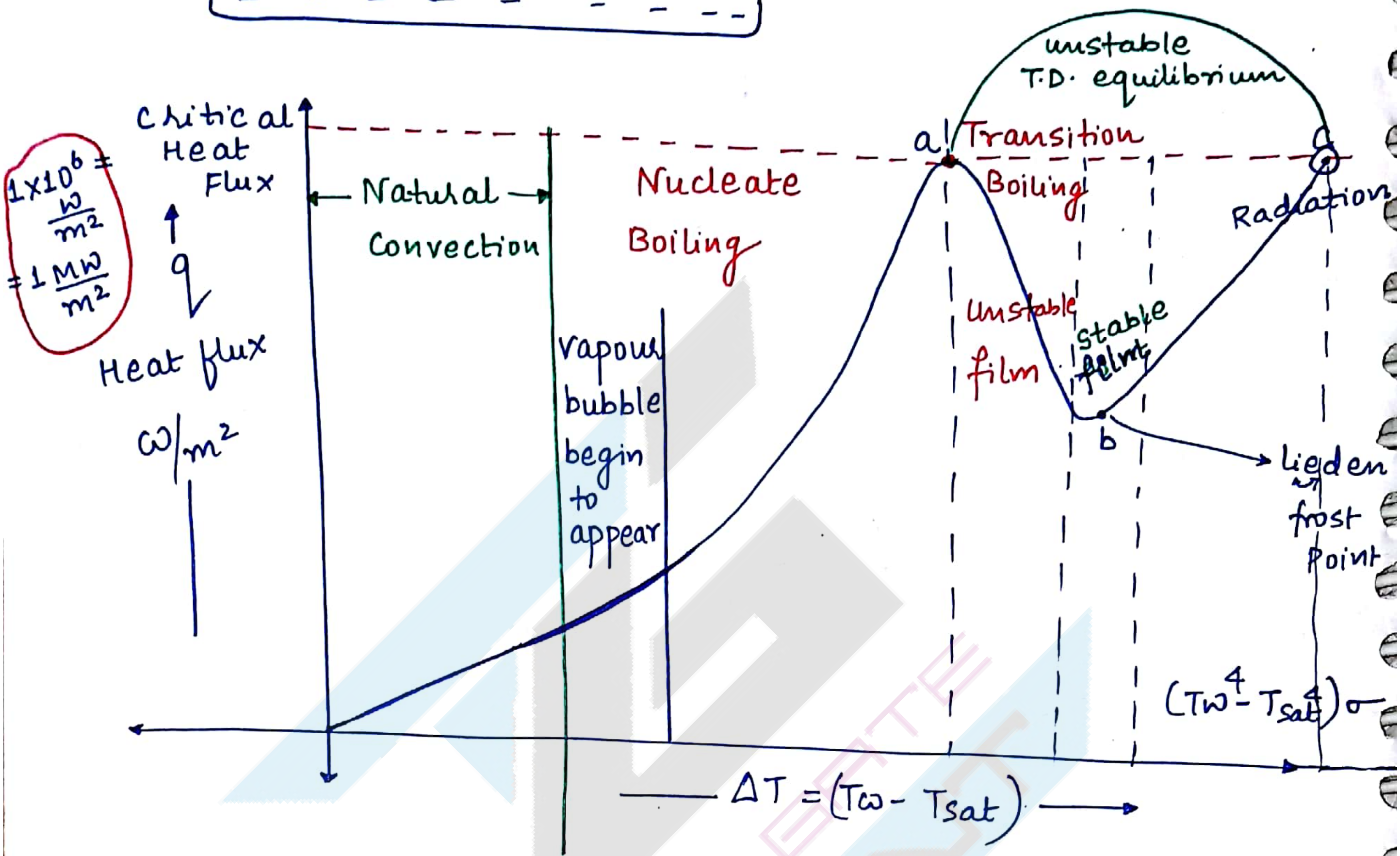

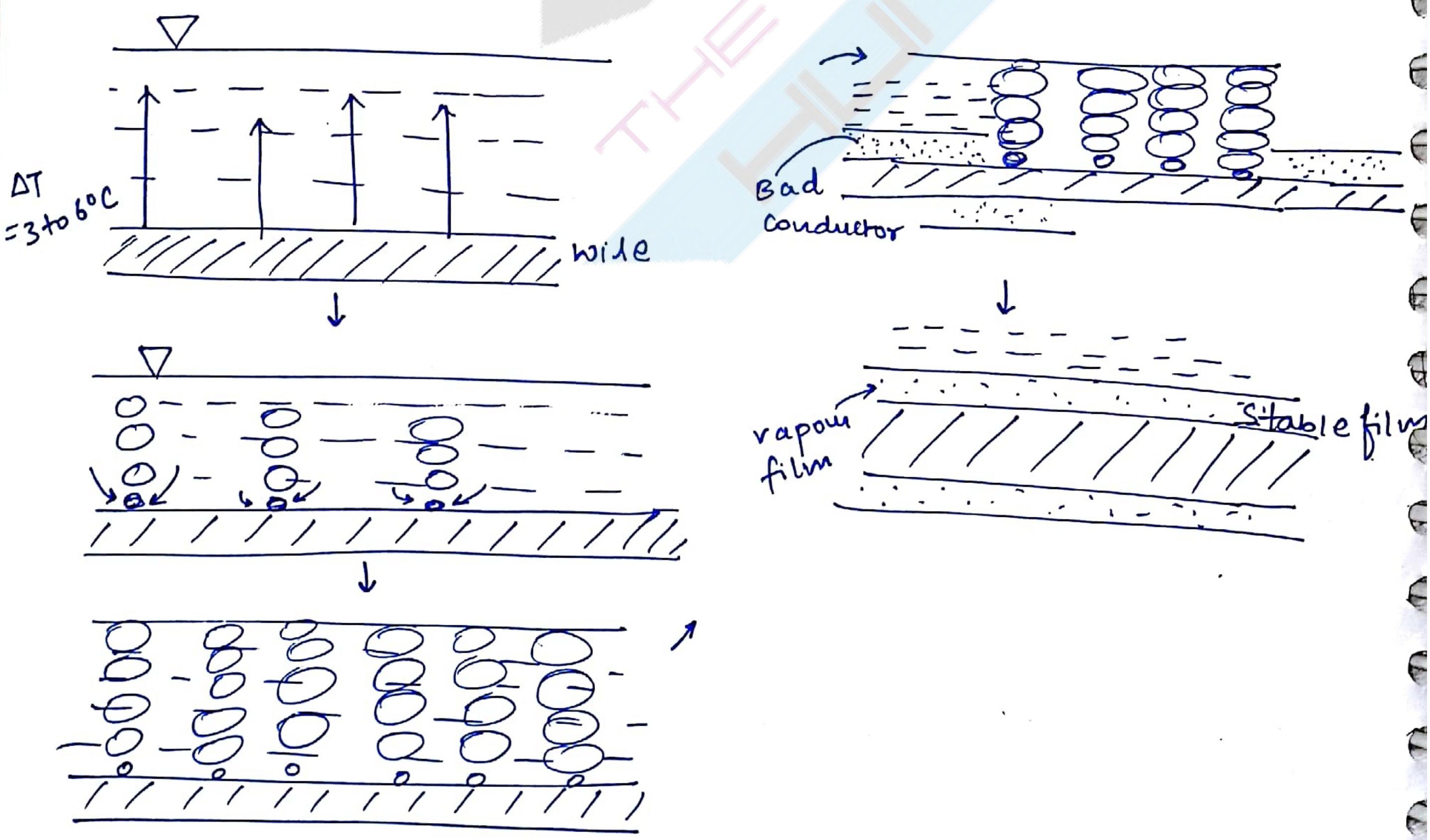

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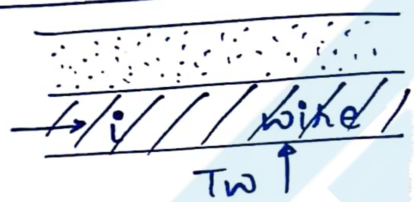
Beyond point (a)



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