LECTURE NOTES

ON

THEORY OF MACHINE

B.TECH
(MECHANICAL ENGINEERING)

FORCE ANALYSIS & GYROSCOPE

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PRECISIONAL MOTION AND FORCE ANALYSIS

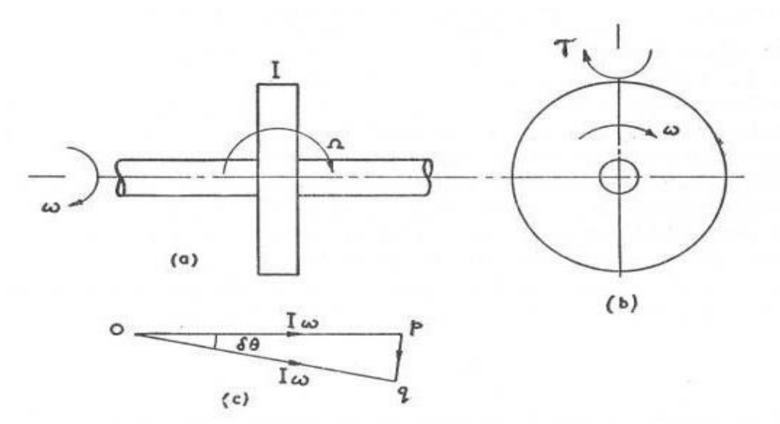
Introduction

Whilst Gyroscopes are used extensively in aircraft instrumentation and have been utilised in monorail trains, the everyday impact of gyroscopic forces on our lives is unappreciated and significant. The simple example is a child's top which would not work but for the gyroscopic couple which keeps it upright. On a slightly different level, the gyroscopic couple helps us to keep a bicycle upright. It is interesting and instructive to remove a bicycle wheel from its frame, hold it by the axle, spin the wheel and then try to change the orientation of the axle. The force required to do so is considerable! However these gyroscopic forces are not always beneficial and it will be shown that the effect on the wheels of a car rounding a corner is to increase the tendency for the vehicle to turn over.

Gyroscopic Couple

Without an understanding of Angular movement it is difficult to understand Gyroscopic Couples. For this reason the Paragraph on Angular Displacement; Velocity and Acceleration shown in The Theory of Machines - Mechanisms, has been reproduced here.

If a uniform disc of polar moment of inertia **I** is rotated about its axis with an angular velocity ω, its **Angular Momentum** $I\omega$ is a vector and can be represented in diagram (c), by the line op which is drawn in the direction of the axis of rotation. The sense of the rotation is clockwise when looking in the direction of the arrow.



If now the axis of rotation is **precessing** with a uniform angular velocity Ω about an axis perpendicular to that of , then after a time δt, the axis of rotation will have turned through an angle $\delta\theta = \Omega t$and the momentum vector will be oq. The **Gyroscopic Couple** is given by:-

$$\tau = \text{The rate of change of angular momentum} = \frac{pq}{\delta t}$$

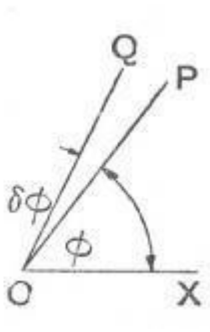
$$= \frac{I\,\omega\,\delta\theta}{\delta t} = I\,\omega\,\Omega \quad \text{In the limit}$$

- The direction of the couple acting on the gyroscope is that of a clockwise rotation when looking in the direction pq.
- In the limit the direction of the couple is perpendicular to the axe of both ω and Ω
- The reaction couple exerted by the gyroscope on its frame is in the **reverse sense**(It is advisable to draw the vector triangle opq in each case.

Angular Displacement, Velocity And Acceleration

Let:- The line OP in the diagram rotates around O

- Its inclination relative to OX be ϕ radians.



Then if after a short period of time the line has moved to lie along OQ, then the angle $\delta\theta$is **The Angular Displacement** of the line.

- **Angular Displacement** is a vector quantity since it has both magnitude and direction.

Angular Displacement:

In order to completely specify and angular displacement by a vector, the vector must fix:-

- The direction of the axis of rotation in space.
- The sense of the angular displacement. i.e. whether clockwise or anti-clockwise.

- The magnitude of the angular displacement.

  In order to fix the vector can be drawn at right angles to the plane in which the angular displacement takes place, say along the axis of rotation and its length will be , to a convenient scale, the magnitude of the displacement.

  The conventional way of representing the sense of the vector , is to use the right-hand screw rule. i.e,

- The arrow head points along the vector in the same direction as a right handed screw would move, relative to a fixed nut.
- Using the above convention, the angular displacement $\delta\theta$shown in the diagram would be represented by a vector perpendicular to the plane of the screen and the arrow head would point away from the screen.

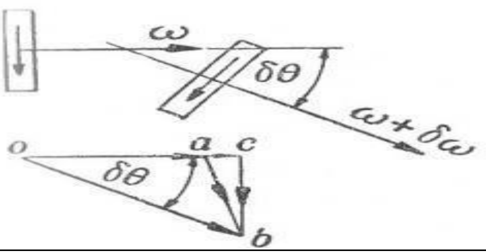
Angular Velocity:

- This is defined as the rate of change of angular displacement with respect to time.
- As angular velocity has both magnitude and direction it is a**vector quantity** and may be represented in the same way as angular displacement.
- If the direction of the angular displacement vector is constant. i.e.The plane of the angular displacement does not change its direction,. Then the angular velocity is merely the change in magnitude of the angular displacement with respect to time.

Angular Acceleration:

- Defined as the rate of change of angular velocity with respect to time.
- A Vector quantity.
- The direction of the acceleration vector is not necessarily the same as the displacement and velocity vectors.

Assume that a given instant a disc is spinning with an angular velocity of ω in a plane at right angles to the screen and that after a short interval of δtits speed has increased to $\omega + \delta\omega$.



Then applying the right-hand rule:-

- The angular velocities at the two instants are represented by the vectors **oa** and **ob**
- The change of angular velocity in a time of δt is represented by the vector **ab**. This can be resolved into two components **ac** and **cb** which are respectively parallel and perpendicular to **oa**

- The component parallel to oa is given by:- $\alpha_T = \frac{d\omega}{dt}$
- The component perpendicular to oa is given by $\alpha_C = \omega \frac{d\theta}{dt} = \omega \times \omega_P$

The subject Dynamics of Machines may be defined as that branch of Engineering-science, which deals with the study of relative motion between the various parts of a machine, and forces which act on them. The knowledge of this subject is very essential for an engineer in designing the various parts of a machine. A machine is a device which receives energy in some available form and utilises it to do some particular type of work. If the acceleration of moving links in a mechanism is running with considerable amount of linear and/or angular accelerations, inertia forces are generated and these inertia forces also must be overcome by the driving motor as an addition to the forces exerted by the external load or work the mechanism does.

NEWT ON' S LAW :

First Law:

Everybody will persist in its state of rest or of uniform motion (constant velocity) in a straight line unless it is compelled to change that state by forces impressed on it. This means that in the absence of a non-zero net force, the center of mass of a body either is at rest or moves at a constant velocity.

Second Law

A body of mass *m* subject to a force **F** undergoes an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force and inversely proportional to the mass, i.e., **F** = *m***a**. Alternatively, the total force applied on a body is equal to the time derivative of linear momentum of the body.

Third Law

The mutual forces of action and reaction between two bodies are equal, opposite and collinear. This means that whenever a first body exerts a force **F** on a second body, the second body exerts a force **−F** on the first body. **F** and **−F** are equal in magnitude and opposite in direction. This law is sometimes referred to as the *action-reaction law*, with **F** called the "action" and **−F** the "reaction".

Principle of Super Position:

Sometimes the number of external forces and inertial forces acting on a mechanism are too much for graphical solution. In this case we apply the method of superposition. Using superposition the entire system is broken up into (n) problems, where n is the number of forces, by considering the external and inertial forces of each link individually. Response of a linear system to several forces acting simultaneously is equal to the sum of responses of the system to the forces individually. This approach is useful because it can be performed by graphically.

Free Body Diagram:

A free body diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem. The diagrams are also used as a conceptual device to help identify the internal forces—for example, shear forces and bending moments in beams—which are developed within structures.

DYNAMIC ANALYSIS OF FOUR BAR MECHANISM:

A **four-bar linkage** or simply a **4-bar** is the simplest movable linkage. It consists of four rigid bodies (called bars or links), each attached to two others by single joints or pivots to form closed loop. Four- bars are simple mechanisms common in mechanical engineering machine design and fall under the study of kinematics.

D-Alembert's Principle

Consider a rigid body acted upon by a system of forces. The system may be reduced to a single resultant force acting on the body whose magnitude is given by the product of the mass of the body and the linear acceleration of the centre of mass of the body. According to Newton's second law of motion,

$F = m.a$

F = Resultant force acting on the body,

m = Mass of the body, and

= Linear acceleration of the centre of mass of the a body.

The equation **(*i*)** may also be written as: $F - m.a = 0$

A little consideration will show, that if the quantity – m.a be treated as a force, equal, opposite and with the same line of action as the resultant force F, and include this force with the system of forces of which F is the resultant, then the complete system of forces will be in equilibrium. This principle is known as D- Alembert's principle. The equal and opposite force – m.a is known as reversed effective force or the inertia force (briefly written as F_I). The equation (ii) may be written as $F + F_I = 0$...(iii)

Thus, D-Alembert's principle states that the resultant force acting on a body together with the reversed effective force (or inertia force), are in equilibrium. This principle is used to reduce a dynamic problem into an equivalent static problem.

Velocity and Acceleration of the Reciprocating Parts in Engines

The velocity and acceleration of the reciprocating parts of the steam engine or internal combustion engine (briefly called as I.C. engine) may be determined by graphical method or analytical method. The velocity and acceleration, by graphical method, may be determined by one of the following constructions: 1. Klien's construction, **2.** Ritterhaus's construction, and **3.** Benett's construction.

SOLVED PROBLEMS

The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction: **1.** Velocity and acceleration of the piston, **2.** Velocity and acceleration of the mid point of the connecting rod, and **3.** Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).

Solution. Given: OC = 200 mm = 0.2 m ; PC = 700 mm = 0.7 m ; ω = 120 rad/s

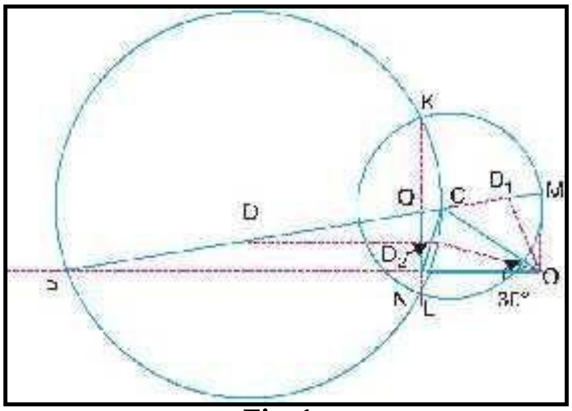


Fig. 1

The Klein's velocity diagram *OCM* and Klein's acceleration diagram *CQNO* as shown in Fig. 1 is drawn to some suitable scale, in the similar way as discussed in Art. 15.5. By measurement, we find that OM = 127 mm = 0.127 m ; CM = 173 mm = 0.173 m ; QN = 93 mm = 0.093 m ; NO = 200 mm

Velocity and acceleration of the piston

We know that the velocity of the piston $P, v_P = \omega \times OM = 120 \times 0.127 = 15.24$ m/s **Ans.**

Acceleration of the piston P, $a_P = \omega^2 \times NO = (120)^2 \times 0.2 = 2880$ m/s^2 **Ans.**

Velocity and acceleration of the mid-point of the connecting rod

In order to find the velocity of the mid-point D of the connecting rod, divide CM at D_1 in the same ratio as D divides CP. Since D is the mid-point of CP, therefore D_1 is the mid-point of CM, *i.e.* $CD_1 = D_1M$. Join OD_1. By measurement, OD_1 = 140 mm = 0.14 m

Velocity of D, $v_D = \omega \times OD_1 = 120 \times 0.14 = 16.8$ m/s **Ans.**

In order to find the acceleration of the mid-point of the connecting rod, draw a line DD_2 parallel to the line of stroke PO which intersects CN at D_2. By measurement,

OD_2 = 193 mm = 0.193 m

∴ Acceleration of D,

$a_D = \omega^2 \times OD_2 = (120)^2 \times 0.193 = 2779.2$ m/s^2 **Ans.**

1. Angular velocity and angular acceleration of the connecting rod

We know that the velocity of the connecting rod PC (*i.e.* velocity of P with respect to C), $v_{PC} = \omega \times CM = 120 \times 0.173 = 20.76$ m/s

EQUIVALENT DYNAMICAL SYSTEM

In order to determine the motion of a rigid body, under the action of external forces, it is usually convenient to replace the rigid body by two masses placed at a fixed distance apart, in such a way that,

1. the sum of their masses is equal to the total mass of the body ;
2. the centre of gravity of the two masses coincides with that of the body ; and
3. the sum of mass moment of inertia of the masses about their centre of gravity is equal to the mass moment of inertia of the body.

When these three conditions are satisfied, then it is said to be an ***equivalent dynamical system.***

Consider a rigid body, having its centre of gravity at *G*,

Let m = Mass of the body, k_G = Radius of gyration about its centre of gravity *G*,

m_1 and m_2 = Two masses which form a dynamical equivalent system,

CORRECTION COUPLE TO BE APPLIED TO MAKE TWO MASS SYSTEM DYNAMICALLY EQUIVALENT

In Art.2 , we have discussed the conditions for equivalent dynamical system of two bodies. A little consideration will show that when two masses are placed arbitrarily*, then the conditions (*i*) and (*ii*) as given in Art. 2 will only be satisfied. But the condition (*iii*) is not possible to satisfy. This means that the mass moment of inertia of these two masses placed arbitrarily, will differ than that of mass moment of inertia of the rigid body.

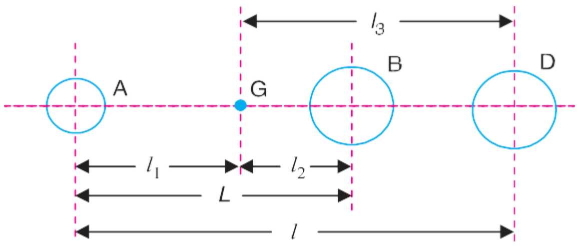


Fig. 2. Correction couple to be applied to make the two-mass system dynamically equivalent.

INERTIA FORCES IN A RECIPROCATING ENGINE, CONSIDERING THE WEIGHT OFCONNECTING ROD

In a reciprocating engine, let *OC* be the crank and *PC*, the connecting rod whose centre of gravity lies at *G*. The inertia forces in a reciprocating engine may be obtained graphically as discussed below:First of all, draw the acceleration diagram *OCQN* by Klien's construction. We know that the acceleration of the piston *P* with respect to *O*,

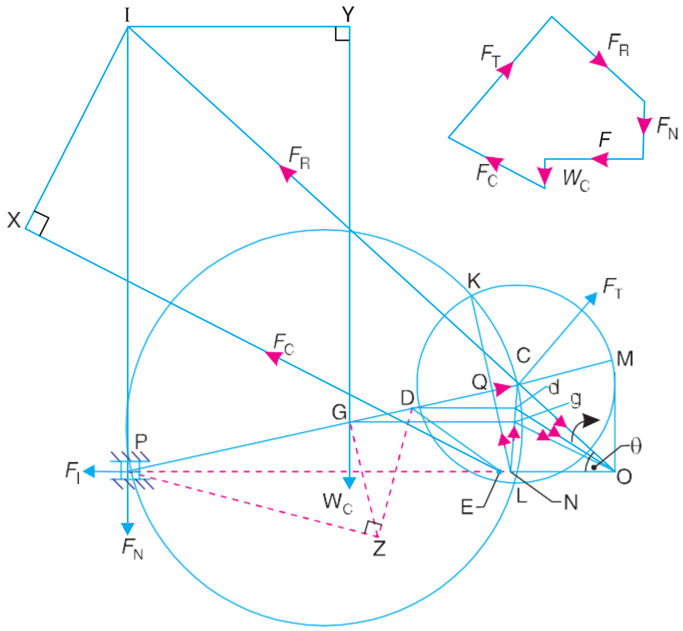


Fig. 3. Inertia forces is reciprocating engine, considering the weight of connecting rod.

Replace the connecting rod by dynamically equivalent system of two masses as discussed in Art. 15.12. Let one of the masses be arbitrarily placed at *P*. To obtain the position of the other mass, draw *GZ* perpendicular to *CP* such that *GZ* = *k*, the radius of gyration of the connecting rod. Join *PZ* and from *Z* draw perpendicular to *DZ* which intersects *CP* at *D*. Now, *D* is the position of the second mass.

Note: The position of the second mass may also be obtained from the equation,

$GP \times GD = k^2$

Locate the points *G* and *D* on *NC* which is the acceleration image of the connecting rod. This is done by drawing parallel lines from *G* and *D* to the line of stroke *PO*. Let these

From *D*, draw *DE* parallel to *dO* which intersects the line of stroke *PO* at *E*. Since the accelerating forces on the masses at *P* and *D* intersect at *E*, therefore their resultant must also pass through *E*. But their resultant is equal to the accelerang force on the rod, so that the line of action of the accelerating force on the rod, is given by a line drawn through *E* and parallel to *gO*, in the direc- tion from *g* to *O*. The inertia force of the connecting rod F_C therefore acts through *E* and in the opposite direction.

A little consideration will show that the forces acting on the connecting rod are :

Inertia force of the reciprocating parts (F_I) acting along the line of stroke *PO*,The side thrust between the crosshead and the guide bars (F_N) acting at P and right angles to line of stroke PO,

The weight of the connecting rod($W_C = m_C.g$),

Inertia force of the connecting rod (F_C),

(a) The radial force (F_R) acting through O and parallel to the crank OC,

(b) The force (F_T) acting perpen- dicular to the crank OC.

Now, produce the lines of action of F_R and F_N to intersect at a point I, known as instantaneous centre. From I draw IX and IY, perpendicular to the lines of action of F_C and W_C. Taking moments about I,

$$F_T \times IC = F_I \times IP + F_C \times IX + W_C \times IY \qquad \text{...}(ii)$$

The value of F_T may be obtained from this equation and from the force polygon as shown in Fig. 15.22, the forces F_N and F_R may be calculated. We know that, torque exerted on the crankshaft to overcome the inertia of the moving parts = $F_T \times OC$

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